OPERATIONAL RESEARCH AND MATHEMATICAL PROGRAMMING

Syllabus

Requisites of the Course

Cycle of Higher Education	***Second (Master's)***
Field of Study	*12 Information technologies*
Speciality	*121 Software Engineering*
Educational Program	*Software Engineering of Multimedia and Information Retrieval Systems*
Type of Course	*Normative*
Mode of Studies	*Full-time*
Year of studies, semester	*2nd year, autumn semester*
ECTS workload	*Lectures: 54 hours, computer workshop: 18 hours, laboratory classes: 36 hours, independent work: 102 hours.*
Testing and assessment	*Exam, modular control work, calendar control*
Course Schedule	*According to the schedule for the autumn semester of the current academic year (rozklad.kpi.ua)*
Language of Instruction	*English*
Course Instructors	Lecturer: Ph.D. *, Associate Professor, Onai Mykola Volodymyrovych, onay@pzks.fpm.kpi.ua* Computer workshop and laboratory classes: Ph.D. *, Associate Professor, Onay Mykola Volodymyrovych, onay@pzks.fpm.kpi.ua*

Outline of the Course

1. Course description, goals, objectives, and learning outcomes

The study of the discipline "Operational Research and Mathematical Programming" allows students to form the competencies needed to solve practical problems of professional activities related to solving complex economic and mathematical models.

The discipline "Operational Research and Mathematical Programming" ensures the successful completion of a master's thesis and the acquisition of knowledge and individual tasks, while continuing postgraduate studies in various natural sciences.

The purpose of *studying the discipline "Operational Research and Mathematical Programming" is the formation of students' ability to build and effectively apply economic and mathematical models, analyze the constructed mathematical model, solve problems arising in the process of modeling by linear, nonlinear, dynamic programming and numerical optimization methods.*

The subject of the *discipline "Operational Research and Mathematical Programming" are methods of building of mathematical models and solving optimization problems.*

The study of the discipline (credit module) "Operations Research and Mathematical Programming" allows students to form the professional competencies, which are necessary to solve practical problems of professional activity:

PC11	*Ability to apply and develop fundamental and interdisciplinary knowledge to successfully solve scientific problems of software engineering.*
PC20	*Ability to develop and apply methods and algorithms for making optimal decisions, solve complex optimization problems using software tools.*

Program learning outcomes of the discipline (credit module) "Operations Research and Mathematical Programming":

PLO03	*Build and research models of information processes in the application field.*
PLO06	*Develop and evaluate software design strategies; substantiate, analyze and evaluate options for design solutions in terms of the quality of the final software product, resource constraints and other factors.*
PLO12	*Make effective organizational and managerial decisions in conditions of uncertainty and changing requirements, compare alternatives, assess risks.*
PLO14	*Predict the development of software systems and information technology.*
PLO17	*Collect, analyze, evaluate the information needed to solve scientific and applied problems, using scientific and technical literature, databases and other sources.*
PLO18	*To develop mathematical and programming software for scientific research in the field of software engineering.*
PLO21	*Know the theoretical foundations underlying research methods of information systems and software, research methodologies and computational experiments.*
PLO35	*Know the software methods of operations research and mathematical programming.*

2. Prerequisites and post-requisites of the course (the place of the course in the structural-logical scheme of studies in accordance with educational program)

Successful study of the discipline "Operational Research and Mathematical Programming" is preceded by the study of the discipline "Methodology of Software Engineering" from the curriculum for masters in the specialty 121 Software Engineering.

The theoretical knowledge and practical skills obtained as a result of mastering the discipline "Operational Research and Mathematical Programming" can be useful for conducting scientific research on the topic of the dissertation.

3. Content of the course

The discipline "Operational Research and Mathematical Programming" involves the study of the following topics:

Topic 1. Elements of classical optimization theory

Topic 2. Linear programming

Topic 3. General provisions and basic concepts of Operational research

Topic 4. Transport problems of linear programming

Topic 5. Nonlinear programming

Topic 6. Numerical methods of nonlinear optimization

Topic 7. Dynamic programming

Topic 8. Systems of mass service

Topic 9. Optimizational methods in large-scale problems

Topic 10. Elements of game theory

Modular control work

Exam

4. Teaching materials and resources

Basic literature:

1. *Yatsko O.M., Tomka Yu.Ia. Doslidzhennia operatsii ta teoriia ihor: Navchalnyi-metodychnyi posibnyk. – Chernivtsi: Vyd-vo Tekhnodruk, 2023. – 392 p.*
2. *Yu.P. Зайченко. Operational Research. Textbook. Seventh edition, revised and supplemented. - Kyiv: Slovo Publishing House, 2006. - 816 p.*
3. *Zaichenko O.Yu., Zaichenko Yu.P. Operational Research. Collection of tasks. - Kyiv: Slovo Publishing House , 2007. - 472 p.*

Additional literature:

4. *Hamdy Taha Operations Research: An Introduction, Pearson; 11th edition (July 6, 2022). – 848 pages.*
5. *Wayne L. Winston Operations Research: Applications and Algorithms Cengage Learning; 4th edition. – 1440 pages*
6. *David Mautner Himmelblau Applied Nonlinear Programming. – McGraw-Hill; 1st Edition .*
7. *von Neumann, John; Morgenstern, Oskar* Theory of games and economic behaviour - *3rd ed. Princeton, New Jersey: Princeton University Press*
8. *J.S, Sharma Operations Research: Theory and Aplications. Sixth Edition – Trinity Press, 6th edition*
9. *Duffin, Richard J.* Geometric programming - *theory and application New York [etc.]: John Wiley & Sons.*

Educational content

5. Methodology of mastering the discipline (educational component)

№	*Type of training lesson*	*Description of training lesson*
	Topic 1. Elements of classical optimization theory	
1	*Lecture 1. Necessary and sufficient conditions of conditional extremum*	*The first necessary condition for the existence of an extremum. Stationary and critical points. The second necessary condition for the existence of an extremum. Sufficient condition for the existence of an extremum. Saddle point of function. Scheme of finding an extremum. Function level line. Function level hyperplane. Derived in the direction of the level line. Lagrange method. Geometric interpretation of the Lagrange method. Sufficient conditions for determining an extreme point. The circled Hesse matrix. Hesse matrix with parameter. Active and passive*

		restrictions. Necessary conditions of Kuhn-Tucker. Sufficiency of Kuhn-Tucker conditions. Sufficient conditions for the existence of a conditional extremum.
2	*Laboratory class 1*	*Hesse matrix with parameter. Active and passive restrictions. Necessary conditions of Kuhn-Tucker. Sufficiency of Kuhn-Tucker conditions. Sufficient conditions for the existence of a conditional extremum.*
3	*Lecture 2. Saddle point of the Lagrange function*	*Determination of the saddle point of the Lagrange function. Slater's condition of regularity. Kuhn-Tucker theorem for the saddle point of the Lagrange function.*
4	*Computer workshop 1*	*Determination of the saddle point of the Lagrange function*
	Topic 2. Linear programming	
5	*Lecture 3. Problem statement and basic provisions*	*The general problem of linear programming. Standard linear programming problem. Canonical problem of linear programming. Transition from one form of linear programming problem to another. Convex linear combination of vectors. Inner and boundary point. Closed set. Polyhedron of solutions. Angular point of the polyhedron of solutions. Admissible basic solution. Degenerate plan.*
6	*Lecture 4. Graphic method for solving linear programming problems*	*Graphical representation of the set of admissible solutions. The main stages of solving the problem of linear programming by graphical method. The condition for the feasibility of using the graphical method.*
7	*Laboratory class 2*	*Solving the problem of linear programming by graphical method.*
8	*Lecture 5. Ordinary simplex method and related methods*	*Finding the solution of the linear programming problem by the search method. Determining the total number of reference plans. Definition of simplex, n-simplex and standard n-simplex. Simplicial polyhedron. Ellipsoid algorithm. Internal point method.*
9	*Computer workshop 2*	*Ordinary simplex method.*
10	*Lecture 6. Tabular simplex method*	*Simplex-difference. Jordan-Gaussian’s transformation. Simplex transformation. Algorithm for solving the problem of linear programming by the tabular simplex method.*
11	*Laboratory class 3*	*Tabular simplex method*
12	*Lecture 7. Artificial basis method*	*Determining the types of tasks for which it is necessary to use the method of artificial basis. The strategy of introducing artificial coefficients into the target function. Advanced problem of linear programming.*
13	*Laboratory class 4*	*Artificial basis method*

14	*Lecture 8. Two-stage simplex method*	*Overview of tasks for which it is not advisable to use the method of artificial basis. The first stage of the two-stage simplex method. The second stage of the two-stage simplex method. The content of artificial variables.*
15	*Laboratory class 5*	*Two-stage simplex method*
16	*Lecture 9. Elements of the theory of duality*	*Formulation of the Kuhn-Tucker theorem for the linear programming problem. Conditions for the existence of a saddle point for the problem of linear programming. The relationship between the problem of maximization and minimization. Direct and dual problem of linear programming on the example of economic-mathematical model. Objectively determined estimates of the resource. The basic duality theorem. Complementary non-rigidity theorem in linear programming problems. Theorem of the existence of an optimal solution. Symmetric and asymmetric linear programming problems.*
Topic 3. General provisions and basic concepts of Operational research		
17	*Lecture 10. Classification of extreme problems and the main stages of their solution*	*Inventory management problems. Resource allocation problems. Problems of mass service. Calendar planning problems.*
Topic 4. Transport problems of linear programming		
18	*Lecture 11. Statement, basic properties of the transport problem and methods of finding the initial basic solution*	*Formulation of the problem. Transportation plan. Transport problem as a problem of linear programming. Dual transport problem. Transport task with limited bandwidth capacities. The method of the north-west angle to find the initial basic solution. The minimum cost method for finding the initial basic solution. Vogel's method for finding the initial basic solution.*
19	*Computer workshop 3*	*Finding the initial reference plan of the transport problem. Statement of the problem. Transportation plan. Transport problem as a problem of linear programming. Dual transport problem. Transport task with limited bandwidth capacities. The method of the north-west angle to find the initial basic solution. The minimum cost method for finding the initial basic solution. Vogel's method for finding the initial basic solution.*
20	*Lecture 12. Method of potentials and Hungarian method*	*Sufficient conditions for the optimality of the allowable solution. Estimated values. Algorithm №1 for implementation of the method of potentials. Algorithm №2 for implementation of the of the method of potentials. Conversion cycle. Relationship of the of the method of potentials with the simplex method. Kuhn-*

		Mancres algorithm. Two forms of representation of the general Hungarian algorithm. Task of appointment.
21	*Laboratory class 6*	*Method of potentials*
22	*Computer workshop 4*	*The method of potentials for solving the transport problem. Sufficient conditions for the optimality of the allowable solution. Estimated values. Algorithm №1 for implementation of the method of potentials. Algorithm №2 for implementation of the of the method of potentials. Conversion cycle. Relationship of the of the method of potentials with the simplex method.*
23	*Laboratory class 7*	*Hungarian method*
24	*Computer workshop 5*	*Hungarian method for solving the transport problem. Kuhn-Mancres algorithm. Two forms of representation of the general Hungarian algorithm. Task of appointment.*
Topic 5. Nonlinear programming		
25	*Lecture 13. Quadratic programming*	*Quadratic form in matrix-vector form. Statement of the quadratic programming problem. Kuhn-Tucker conditions for quadratic programming problems.*
26	*Laboratory class 8*	*Quadratic programming*
27	*Computer workshop 6*	*Methods of solving quadratic programming problems.* *Quadratic form in matrix-vector form. Statement of the quadratic programming problem. Kuhn-Tucker conditions for quadratic programming problems.*
28	*Lecture 14. Geometrical and fractional-linear programming*	*Generalized monomial. Posynomial. Condition of normalization. Pre-dual function. The degree of complexity of the problem of geometric programming. Fractional-linear programming.*
29	*Laboratory class 9*	*Geometrical programming*
30	*Laboratory class 10*	*Fractional-linear programming*
Topic 6. Numerical methods of nonlinear optimization		
31	*Lecture 15. Gradient methods and methods of variable metrics*	*The fastest descent method. Choosing the optimal step size. Newton's method. The method of conjugate directions. Relationship between the optimization problem and the problem of solving a system of nonlinear equations. Broyden's method. David-Fletcher-Powell method.*
32	*Laboratory class 11*	*Gradient methods*
33	*Laboratory class 12*	*Methods of variable metrics*
34	*Lecture 16. Rosen gradient projection*	*The concept of design matrix. Algorithm of gradient projection method for linear constraints. Generalized*

	method, consolidated gradient method and penalty functions method	*consolidated gradient method. Parametric methods. Barrier surface method. Method of penalty functions.*
Topic 7. Dynamic programming		
35	*Lecture 17. Minimization of non-smooth functions*	*Dynamic inventory management prob;em. Subgradientof the function. Chebyshev approximation. Generalized gradient method.*
36	*Laboratory class 13*	*Minimization of non-smooth functions*
37	*Lecture 18. The main idea and features of the computational method of dynamic programming. Dynamic programming for problems with multiple constraints and variables*	*General view of the objective function of the dynamic programming problem. The concept of globally optimal solution. The problem of choosing a trajectory. The task of consistent decision making. Problem with two control variables. Methods for reducing the dimensionality of dynamic programming problems.*
38	*Laboratory class 14*	*Dynamic programming*
39	*Lecture 19. Dynamic problems of inventory management and dynamic programming for Markov processes*	*The problem of inventory management in deterministic stationary demand. The task of managing multi-item stocks while limiting the capacity of the warehouse. Inventory management model for probabilistic demand. The concept of integral discount effect. Methods for estimating infinite sequences of effects. Infinite planning period. Properties of the Markov system. Bellman functional equations.*
40	*Modular control work*	
Topic 8. Queuing systems		
41	*Lecture 20. Basic concepts of the theory of mass service*	*The main components of queuing systems models. Inbound flow of requirements. Exponential distribution in the queuing systems. Poisson flow modeling. Queue organization. Output flow of requirements. Types of models of queuing systems.*
42	*Laboratory class 15*	*Basics of the queuing theory*
43	*Lecture 21. Models of birth and death and the general model of the system of mass service*	*Little's formula. The law of conservation of a stationary queue. Single-channel queuing systems. Multi-channel queuing systems. Generalized algorithm for modeling of queuing systems.*
Topic 9. Optimization methods in large-scale problems		

44	*Lecture 22. Decomposition method*	*Danzig-Wolf decomposition method. The principle of decomposition. The limited coordinating task. Decomposition based on the separation of variables. Benders variables separation method. Decomposition methods based on aggregation.*
45	*Laboratory class 16*	*Decomposition method*
46	*Lecture 23. Decomposition of Kornai-Liptak*	*Restrictions of block-diagonal structure. Brown's fictitious game method. An algorithm that implements the Kornai-Liptak decomposition method*
Topic 10. Elements of game theory		
47	*Lecture 24. Basic concepts and definitions in the field of game theory. Positional games*	*Topological tree. Eagle game. Strategy of the game. Normal form of the game. Game decomposition. Antagonistic or strictly competitive games. Game with complete information. Saddle point of the matrix. Positional games. Party. An absolutely defined game.*
48	*Laboratory class 17*	*Positional games*
49	*Lecture 25. J. von Neumann's minimax theorem*	*The concept of mixed player strategy. The lower price of the game. The top price of the game. Minimax theorem.*
50	*Lecture 26. Application of linear programming for solving matrix games*	*Reduction of matrix game to the problem of linear programming. Solving of the games with a matrix of 2 x n and m x 2. Solving an arbitrary game m x n.*
51	*Laboratory class 18*	*Application of linear programming for solving matrix games*
52	*Computer workshop 7*	*Graphical-analytical method for solving matrix games. Topological tree. Eagle game. Strategy of the game. Normal form of the game. Game decomposition. Antagonistic or strictly competitive games. Game with complete information. Saddle point of the matrix. Positional games. Party. An absolutely defined game.*
53	*Computer workshop 8*	*Analytical method for solving matrix and bimatrix games. The concept of mixed player strategy. The lower price of the game. The top price of the game. Minimax theorem.* *Reduction of matrix game to the problem of linear programming. Solving of the games with a matrix of 2 x n and m x 2. Solving an arbitrary game m x n.*
54	*Lecture 27. Application of linear programming*	*Reduction of a matrix game to a linear programming problem. Solution of an arbitrary game m x n.*

	for solving matrix games of special structure	

6. Independent work of the student

The discipline "Operational Research and Mathematical Programming" is based on independent preparation for classroom classes on theoretical and practical topics.

№ s / n	*The name of the topic that is submitted for independent study*	*Number of hours*	*Literature*
1	*Preparation for lectures*	*27*	*1, 4*
2	*Preparation for computer workshops*	*16*	*1-5*
3	*Preparation for laboratory classes*	*18*	*1-5*
4	*Preparation for modular control work*	*11*	*1-5*
5	*Exam preparation*	*30*	*1-5*

Policy and Assessment

7. Policy of the academic discipline (educational component)

Attendance at computer workshops can be sporadic – if necessary to protect the works of the computer workshop.

Rules of conduct in the classes: activity, respect for those present, turning off the phones.

Adherence to the policy of academic integrity.

Rules for the protection of computer workshop works: the works must be done according to the card of the student, which is determined pseudo-randomly by the pseudo-random number generator (www.random.org) at the beginning of the semester.

The rules for assigning incentive and penalty points are as follows.

Incentive points are awarded for:
- accurate and complete answers during surveys based on lecture materials. During the semester there is a ***blitz poll*** *on the topics of past lectures. Maximum number of points for the blitz poll: 3 points.*
- creative approach in performing computer workshop. The maximum number of points for all works is 2 points.

Penalty points are awarded for:
- plagiarism (the program code does not correspond to the task variant, the identity of the program code among different works) in the works of the computer workshop: -5 points for each attempt.

8. Types of control and rating system of assessment of learning outcomes (RSO)

During the semester students perform 8 computer workshops. Maximum number of points for each computer workshop: 3 points.

Points are awarded for:
- quality of performing of computer workshop: 0-1 points;
- answer to theoretical questions during the defense of the computer workshop: 0-1 points;
- timely submission of work for defense: 0-1 points.

Quality of performance evaluation criteria:
1 point - the work is performed qualitatively, in full;

0 points - the work is not performed in full, or contains significant errors.

Answer's evaluation criteria:
1 point - the answer is complete, well-argued;
0 points - no answer or the answer is incorrect.

Criteria for assessing the timeliness of submission of work for defense:
1 point - the work is submitted for defense no later than the specified deadline;
0 points - the work is submitted for defense later than the specified deadline.

Maximum number of points for the implementation and defense of computer workshops:
3 points × 8 computer workshops = 24 points.

*During the semester, during the lectures is **conducted a survey on the topic of the current lesson** . The maximum number of points for the survey that can be obtained during the semester: 2 points.*

*The task for the **modular control work** consists of 3 questions - 2 theoretical and 1 practical. The answer to each theoretical question is evaluated by 8 points, and the answer to the practical question is evaluated by 10 points.*

Criteria for evaluating of each theoretical question of the modular control work:
8 points - the answer is correct, complete, well argued;
7-6 points – the answer contains insignificant errors;
5-1 points - the answer contains significant errors;
0 points - no answer or the answer is incorrect.

Criteria for evaluating the practical question of the modular control work:
10 points - the answer is corect, the calculations have been completed in full;
9-6 points - the answer is corect, but not very well supported by calculations;
5-1 points - the answer contains significant errors;
0 points - no answer or the answer is incorrect.

Maximum number of points for modular control work:
8 points × 2 theoretical questions + 10 points × 1 practical question) = 26 points.

The rating scale for the discipline is equal to:
$R_c = R_{kom.prakt} + R_{MCW}$ *= 24 points + 26 points = 50 points.*

Calendar control: conducted twice a semester as a monitoring of the current state of compliance with the requirements of the syllabus.

At the first attestation (7th week) the student receives "credited" if his current rating is not less than 50% of the maximum number of points (10 points) that the student can receive before the first attestation.
At the second attestation (13th week) the student receives "credited" if his current rating is not less than 50% of the maximum number of points (20 points) that the student can receive before the second attestation.

*Semester control: **exam***

Conditions of admission to semester control:
A necessary condition for admission to the writing of the examination work is the performance and defense of a computer workshop and a semester rating of at least 30 points.

Composition and evaluation criteria of the examination work:

*The task for the **examination work** consists of 2 practical questions. The answer to each practical question is evaluated by 25 points.*

Criteria for evaluating the practical question of the test:

24-25 points - the answer is correct, the calculations are performed in full;
21-23 points - the answer is correct, but not very well supported by calculations;
17-20 points - in general the answer is correct, but has shortcomings;
12-16 points - there are minor errors in the answer;
1-11 points - there are significant errors in the answer;
0 points - no answer or the answer is incorrect.

Maximum number of points for examination work:
25 points × 2 practical questions = 50 points.
The examination component of the rating scale is equal to: RE = 50 points.
The rating scale in the discipline is equal to: R = RC + RE = 50 + 50 points = 100 points.

The total student rating R is defined as the sum of the student's semester rating RC and the RE points obtained at the exam. The grade is assigned according to the value of R according to table 1.

Table 1 of correspondence of rating points to grades on the university scale:

Score	Grade
100-95	Excellent
94-85	Very good
84-75	Good
74-65	Satisfactory
64-60	Sufficient
Less than 60	Fail
Less than 25 or non-compliance of the requirements	Not Graded

9. Additional information on the discipline (educational component)

The list of questions to be submitted for semester control.

Work program of the academic discipline (syllabus):
Is designed by Ph.D., Assoc. Prof., Onai M.V.
Adopted by Computer Systems Software Department (protocol № 8, 22 January 2025)
Approved by the Methodical commission of the Faculty of Applied Mathematics (protocol № 8, 03 February 2025)