Національний технічний університет України
«КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ
імені ІГОРЯ СІКОРСЬКОГО»

Computer Systems Software Department

MATHEMATICAL MODELING OF SYSTEMS AND PROCESSES

Syllabus

Requisites of the Course

Cycle of Higher Education	***Second (master's)***
Field of Study	*12 Information technologies*
Speciality	*121 Software engineering*
Educational Program	*Software Engineering of Multimedia and Information Retrieval Systems*
Type of Course	*Selective*
Mode of Studies	*Full-time*
Year of studies, semester	*1 year of study, autumn semester*
ECTS workload	*Lectures: 36 hours, computer workshop: 18 hours, independent work: 96 hours.*
Testing and assessment	*Exam, modular control work, calendar control*
Course Schedule	*According to the schedule for the spring semester of the current academic year (rozklad.kpi.ua)*
Language of Instruction	*English*
Course Instructors	*Lecturer* : Ph.D. *, associate professor, Онаш Mykola Volodymyrovych, onay @pzks.fpm.kpi.ua* Laboratory classes: *Ph.D. , associate professor, Онаш Mykola Volodymyrovych, onay @pzks.fpm.kpi.ua*

Outline of the Course

1. Course description, goals, objectives, and learning outcomes

Studying the discipline "Mathematical Modeling of Systems and Processes" allows students to develop the competencies necessary for solving practical problems of professional activity related to the solution of complex economic and mathematical models.

The discipline "Mathematical Modeling of Systems and Processes" ensures the successful completion of a master's thesis and the assimilation of knowledge and the performance of individual tasks, while continuing to study at a graduate school in various natural and scientific disciplines.

The purpose of *studying the discipline "Mathematical Modeling of Systems and Processes" is the formation of students' ability to build and effectively apply economic-mathematical models, analyze the constructed mathematical model, solve problems arising in the process of modeling using linear, non-linear, dynamic programming methods and numerical optimization methods.*

The subject of *the discipline "Mathematical Modeling of Systems and Processes" is methods of building mathematical models and solving optimization problems .*

After mastering the discipline "Mathematical modeling of systems and processes", ***the learning outcomes*** *are:*

knowledge:

- *basics of probabilistic methods of analysis and modeling of systems and simulation modeling methods;*
- *methods of analysis of phase portraits of nonlinear systems of differential equations;*

- *general principles of building mathematical models;*
- *population growth models;*
- *competition models;*
- *models of mechanical systems;*
- *models of population interaction*

skill:

- *build mathematical models of real systems and processes;*
- *conduct an analysis of the adequacy of the mathematical model;*
- *build and conduct multi-criteria analysis of models of mechanical systems, models of population growth, competition and population interaction;*
- *justify the choice of computer tools for building a mathematical model;*
- *apply linearization methods to simplify models described by nonlinear systems of differential equations*

experience:

- *construction of mathematical models of real systems and processes;*
- *analysis of the adequacy of the mathematical model;*
- *application of linearization methods to simplify models described by nonlinear systems of differential equations.*

The discipline "Mathematical modeling of systems and processes" strengthens the formation of FC07 and PLO10 .

2. Prerequisites and post-requisites of the course (the place of the course in the structural-logical scheme of studies in accordance with educational program)

The successful study of the discipline "Mathematical Modeling of Systems and Processes" is preceded by the study of the discipline "Mathematical Modeling of Systems and Processes" of the master's training plan in the specialty 121 Software engineering.

The theoretical knowledge and practical skills obtained as a result of mastering the discipline "Mathematical modeling of systems and processes" can be useful for conducting scientific research on the topic of the dissertation.

3. Content of the course

The discipline "Mathematical Modeling of Systems and Processes" involves the study of the following topics:

Topic 1. General provisions and basic concepts of mathematical modeling

Topic 2. Mathematical models described by differential equations of the first order

Topic 3. Mathematical models described by differential equations of higher orders

Topic 4. Mathematical models described by systems of differential equations

Topic 5. Analysis of systems of differential equations and models described by systems of differential equations

Modular control work

Exam

4. Teaching materials and resources

Basic literature:

1. *Dychka, I. A. Mathematical modeling of systems and processes: computer workshop [Electronic resource]: teaching . help for studies specialty 121 "Software engineering", educational program*

"Software engineering of multimedia and information search systems" / I. AND. Dychka, M. IN. Onay , T. M. Swamp ; KPI named after Igor Sikorsky. – Electronic text data (1 file: 2.42 MB). – Kyiv: KPI named after Igor Sikorskyi, 2023. – 128 p. – Title from the screen. https://ela.kpi.ua/handle/123456789/57239

2. Mathematical modeling of systems and technological processes: collective monograph / V. Yu. Shcherban, O. WITH. Kolysko, Yu. Yu. Shcherban, H. IN. Melnyk, M. AND. Kolysko, A. M. Kirichenko. – Kyiv: Fastbind Ukraine LLC , 2023. – 939 p. https://er.knutd.edu.ua/handle/123456789/24075
3. Pavlenko P. M., Filonenko S. F., Cherednikov O. M., Treytyak V.V. Mathematical modeling of systems and processes: teaching . help - K. : NAU, 2017. – 392 p.
4. Bakhrushin V.E. Mathematical foundations of system modeling: Study guide for students. - Zaporizhzhia: Classical Private University, 2009. - 224 p.

Additional literature:

1. Tomas Witelski , Mark Bowen Methods of Mathematical Modeling : Continuous Systems and Differential Equations Springer
2. Edward A. Bender An Introduction that Mathematical Modeling 272 pages
3. Milton Abramowitz & Irene A. Stegun Handbook of Mathematical Functions : with Formulas , Graphs , and Mathematical Tables 1059 pages

Educational content

5. Methodology of mastering the discipline (educational component)

№	Type of training lesson	Description of training lesson
Topic 1. General provisions and basic concepts of mathematical modeling		
1	Lecture 1. Classification of mathematical models and their properties	Physical and mathematical models. Direct and inverse modeling problem. Identification problem. Synthesis task. Models with concentrated and distributed parameters. Linear and non-linear models. Stationary and non-stationary systems. Deterministic and stochastic mathematical models. Continuous and discrete mathematical models. Functional and structural mathematical models. Analytical and simulation mathematical models. Adequacy, simplicity/complexity, potentiality of the mathematical model. Analysis of dimensionality. Checking the order and nature of dependencies . Study of borderline cases. Checking the closedness and correctness of a mathematical problem.
2	Lecture 2. Control of the correctness of mathematical models	Adequacy, simplicity/complexity, potentiality of the mathematical model. Analysis of dimensionality. Checking the order and nature of dependencies . Study of borderline cases. Checking the closedness and correctness of a mathematical problem.

3	*Lecture 3. Dirac's δ-function and Heaviside's function*	*Unit impulse and unit drop. Bell- shaped function . Definition of Dirac's δ-function. Derivation of the main properties of the Dirac δ-function. Impulse characteristics of the system. Heaviside function . The main properties of the Heaviside function and its connection with Dirac's δ-function. Transient characteristics of the system.*
4	*Laboratory class 1*	*Mathematical models described by differential equations of the first order: find analytical and numerical solutions of the proposed differential equations*
5	*Lecture 4. Elements of error theory*	*Classification of errors: task error, method error, computational error. Absolute error. Relative error. Marginal errors. Errors in performing arithmetic operations. Concept and example of unstable problem. Conditionality of the task. The inverse problem of error theory. Statistical approaches to accounting for errors. Errors of machine arithmetic.*
6	*Lecture 5. Modeling technology*	*Formulation of the problem and meaningful statement of the problem. Construction of a conceptual and mathematical model. Choosing a method of solving a mathematical problem. Software implementation of the model on a computer. Checking the adequacy of the model. Organization and planning of experiments. Interpretation of modeling results and decision-making. Designing the research results.*
7	*Laboratory class 2*	*Mathematical models described by differential equations of the first order: find analytical and numerical solutions of the proposed differential equations: construct a field of directions and integral curves, construct a mathematical model of a given process*
Topic 2. Mathematical models described by differential equations of the first order		
8	*Lecture 6. General information about differential equations and methods of their solution*	*Ordinary differential equations and partial differential equations. A field of directions that defines a differential equation. Determination of the isoclines of the differential equation. Solving differential equations by systems of computer mathematics. Mathematical models for problems on mixtures. Mathematical models of temperature fluctuations.*
9	*Lecture 7. Equilibrium points of differential equation solutions and their stability*	*Stationary points of differential equations. Equilibrium solution of a differential equation. Stability and asymptotic stability of the equilibrium point .*
10	*Laboratory class 3*	*Mathematical models described by differential equations of higher orders: to find analytical and numerical solutions of the proposed differential*

		equations of higher orders; construct a field of directions and integral curves
11	*Lecture 8. Models of population growth and harvesting in a logistic population*	*Malthus' model. Verhulst-Pearl model . The potential of the ecological system. Phase diagram of the logistic equation. Analysis of equilibrium points of differential equations describing population growth models. Analysis of the parameters of the harvest model in the logistic population. Phase diagram for the logistic equation with harvest. Construction of a bifurcation diagram for the logistic equation with harvest.*
12	*Laboratory class 4*	*Mathematical models described by differential equations of higher orders: investigate the equilibrium points of differential equations of higher orders; build a mathematical model of a given system or process*
Topic 3. Mathematical models described by differential equations of higher orders		
13	*Lecture 9. Homogeneous and inhomogeneous linear differential equations of higher orders*	*Fundamental system of solutions . General and partial solutions. Vronsky's functional determinant. Characteristic equation. Polynomial from the operator. Classes of problems that reduce to differential equations of higher orders. Solving differential equations of higher orders using computer mathematics systems. Homogeneous and inhomogeneous differential equations of higher orders.*
14	*Lecture 10. Mechanical vibrations*	*Determination of constant damping and attenuation coefficient. Damping and non-damping oscillations. Free oscillations. Harmonic oscillations. Critical fading. Fading . Strong damping. Forced oscillations. The phenomenon of beating. Amplification factor . The phenomenon of resonance. Concept of practical resonance. Correspondence between mechanical and electrical systems.*
15	*Modular control work. Part 1*	
Topic 4. Mathematical models described by systems of differential equations		
16	*Lecture 11. Systems of differential equations, as well as problems that boil down to solving systems of differential equations*	*Reduction of the system of differential equations to one differential equation of higher order. Matrix-valued function. Vronsky's determinant. The fundamental matrix of a linear homogeneous system. Definition of exponential matrix function. Solving a heterogeneous system of differential equations using the exponential matrix function. Examples of problems on mixtures. Examples of problems in which a group of physical bodies interact with each other. The concept of the natural mode of oscillations of a physical system.*

17	*Laboratory lesson 5*	*Analysis of mathematical models described by systems of differential equations: to find analytical and numerical solutions of proposed systems of differential equations; construct a field of directions and phase portraits and conduct their analysis*
Topic 5. Analysis of systems of differential equations and models described by systems of differential equations		
18	*Lecture 12. Linearization of nonlinear systems of differential equations*	*The phase plane of the system of differential equations. Trajectory of the system. Phase portrait of the system. Equilibrium points of systems of differential equations. Autonomous system of differential equations.*
19	*Lecture 13. Classification of equilibrium points of the system of differential equations*	*Degenerate source node. Saddle point. Dicritical node. Focus. Center. Classification of equilibrium points of the system with roll numbers of the matrix. Stability of the type of equilibrium points to small disturbances. The concept of an almost linear system of differential equations. Basic methods of linearization of nonlinear systems of differential equations.*
20	*Laboratory class 6*	*Analysis of mathematical models described by systems of differential equations: to determine the type of interaction of populations according to the proposed system; to build a mathematical model of the interaction of populations given by the option*
21	*Lecture 14. Analysis of phase portraits and stability of nonlinear systems*	*Comparison of the trajectory of a linearized and almost linear system of differential equations. A stable non-degenerate node. Focus. Unstable saddle point. Unstable non-proprietary node. Steady focus. Stable or unstable center.*
22	*Kolmogorov's generalized models*	*Trophic and topical types of interaction. Symbiosis. Commensalism. Predator-prey. Amensalism. Competition. Neutralism. Hause model . Pislow model . Holing's model .*
2 3	*Laboratory class 7*	*Modeling in the system Simulink : build and explore a Simulink differential equation model; build and investigate a Simulink model of a system of differential equations*
24	*Lecture 16. The classic predator-prey model*	*Lotky-Volterra equation . Determination of the type of stationary points of the predator-prey model. Phase portrait of the predator-prey system. Integral curve of the predator- prey system . Periodic fluctuations of the predator and prey population.*
25	*Lecture 17. Model of competition*	*Kolmogorov's model . Intraspecific and interspecific competition. The influence of intraspecific competition on the Lotky-Volterra model . Dominance of intraspecific competition over interspecific*

		competition. Dominance of interspecific competition over intraspecific competition.
26	*Lecture 18. Classification of equilibrium points of the competition model*	*Linearization of the system describing the competition model. Phase portraits around each equilibrium point of the system describing the competition. The concept of a separatrix on a phase portrait. Global phase portrait and its analysis.*
27	*Modular control work. Part 2*	

6. Independent work of the student

The discipline "Mathematical Modeling of Systems and Processes" is based on independent preparation for classroom classes on theoretical and practical topics.

No.	*The name of the topic submitted for independent processing*	*Number of hours*	*Literature*
1	*Preparation for modular control work. Part 1*	*33*	*1-5*
2	*Preparation for modular control work. Part 2*	*33*	*2-7*
3	*Preparation for the exam*	*30*	*1-7*

Policy and Assessment

7. Policy of the academic discipline (educational component)

- *Attending laboratory classes can be occasional and as needed to protect works.*
- *Rules of behavior in classes: activity, respect for those present, turning off phones.*
- *Adherence to the policy of academic integrity.*
- *of the computer workshop : the work must be done according to the option of the student, which is determined pseudorandomly by the generator of pseudorandom numbers (www.random.org) at the beginning of the semester.*
- *The rules for assigning incentive and penalty points are as follows.*

Incentive points are awarded for:
- accurate and complete answers during surveys based on lecture materials. During the semester, there is ***a blitz poll*** *on the topics of past lectures at the lectures. Maximum points for a blitz survey: 3 points.*
- a creative approach in performing the work of the computer workshop. The maximum number of points for all works is 2 points.

Penalty points are calculated for:
- plagiarism (the program code does not correspond to the task variant, the identity of the program code among different works) in the works of the computer workshop: -5 points for each attempt.

8. Types of control and rating system of assessment of learning outcomes (RSO)

During the semester, students perform 8 laboratory works. The maximum number of points for each laboratory work: 3 points.

Points are awarded for:
- quality of performance of the computer workshop: 0-1 points;
- answer to theoretical questions during the defense of the computer workshop: 0-1 points;

- timely presentation of work for defense: 0-1 points.

Performance evaluation criteria:
1 point – the work is done with quality, in full;
0 points – the work is incomplete or contains significant errors.

Answer evaluation criteria:
1 point – the answer is complete, well-argued;
0 points - there is no answer or the answer is incorrect.

Criteria for evaluating the timeliness of work submission for defense:
1 point – the work is submitted for defense no later than the specified deadline;
0 points – the work is submitted for defense later than the specified deadline.

The maximum number of points for performing and defending laboratory work:
3 points × 8 laboratory works = 24 points.

*The task for each **modular test** consists of 3 questions - 2 theoretical and 1 practical. The answer to each theoretical question is worth 2 points, and the answer to a practical question is worth 3 points.*

Evaluation criteria for each theoretical test question:
2 points – the answer is correct, complete, well-argued;
1 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.

Evaluation criteria for the practical test question:
4 points – the answer is correct, the calculations are completed in full;
3-2 points – the answer is correct, but not very well supported by calculations;
1 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.

The maximum number of points for a modular control work:
*2 papers * (2 points × 2 theoretical questions + 4 points × 1 practical question) = 16 points.*

The rating scale for the discipline is equal to:
$R_{c} = R_{com.practice} + R_{interview} + R_{MKR}$ *= 24 points + 16 points = 50 points.*

Calendar control: is carried out twice a semester as a monitoring of the current state of meeting the syllabus requirements .
At the first certification (7th week), the student receives "passed" if his current rating is at least 50% (20 points) of the maximum number of points that the student can receive before the first certification.
At the second certification (13th week), the student receives "passed" if his current rating is at least 50% (35 points) of the maximum number of points that the student can receive before the second certification.

*Semester control: **exam***

Conditions for admission to semester control:
A necessary condition for admission to the examination work is the completion and protection of a computer workshop and a semester rating of at least 30 points.

The composition and evaluation criteria of the examination work:

***exam** task consists of 2 practical questions. The answer to each practical question is evaluated by 25 points.*

Evaluation criteria for the practical test question:
24-25 points – the answer is correct, the calculations are completed in full;
21-23 points - the answer is correct, but not very well supported by calculations;
17-20 points - in general, the answer is correct, but has flaws;
12-16 points – there are minor errors in the answer;
1-11 points – there are significant errors in the answer;

0 points - there is no answer or the answer is incorrect.

The maximum number of points for the examination work:
25 points × 2 practice questions = 50 points.

Table of correspondence of rating points to grades on the university scale :

Number of points	*Rating*
100-95	Perfectly
94-85	Very good
84-75	Good
74-65	Satisfactorily
64-60	Enough
Less than 60	Unsatisfactorily
Less than 30 or admission conditions are not met	Not allowed

9. Additional information on the discipline (educational component)

The list of questions submitted for semester control.

Work program of the academic discipline (syllabus):

Is designed by Ph.D., Assoc. Prof., Onai M.V.

Adopted by Computer Systems Software Department (protocol № 8, 22 January 2025)

Approved by the Methodical commission of the Faculty of Applied Mathematics (protocol № 8, 03 February 2025)