



Algorithmic Support of Multimedia and

Information Retrieval Systems

Working program of the academic discipline (Syllabus)

Details of the academic discipline

Level of Higher Education	First (Bachelor's)	
Field of Study		
•	12 Information technologies	
Specialty	121 Software Engineering	
Educational Program	Software Engineering of Multimedia and Information Retrieval Systems	
Type of Course	Normative	
Mode of Studies	Full-time	
Year of studies, semester	2nd year, 4th semester	
ECTS workload	<i>Lectures: 36 hours, workshop classes: 36 hours, laboratory work: 18 hours, independent work: 90 hours.</i>	
Testing and assessment	Assessment, modular control work, calendar control, calculation work	
Course Schedule	According to the schedule for the spring semester of the current academic year (rozklad.kpi.ua)	
Language of instruction	English	
Course instructors	Lecturer: Ph.D., Associate Professor, Onai Mykola Volodymyrovych, onay@pzks.fpm.kpi.ua Computer workshop and laboratory classes: Ph.D., Associate Professor, Onai Mykola Volodymyrovych, onay@pzks.fpm.kpi.ua	
Placement of the course	Google Classroom Course code sbveayn	

Outline of the Course

1. Course description, goals, objectives, and learning outcomes

Studying the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" allows students to develop the competencies necessary for solving practical problems of professional activity related to data analysis, information processing and the use of modern information-search technologies. The discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" consists of two parts: part 1 "Numerical methods"; part 2 "Mathematical statistics".

The purpose of the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" is to form students of the ability to master the basic terminology of the discipline; to classify standard problems by features, to solve them; apply basic practical techniques for solving standard problems of numerical methods (bisection method, chord method, Newton's method, secant method, chord-tangent method, simple iteration method, LU-factorization method, Kholetsky method, rotation method, reflection method) and mathematical statistics (sampling method, methods of estimating distribution parameters, method of characteristic functions, methods of calculating sample characteristics, estimating sample parameters).

The subject of the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" are methods, algorithms and models used for numerical and statistical data analysis in the development of information search systems.

The study of the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" contributes to the formation of **professional competences (PC)** necessary for solving practical tasks of professional activity related to the analysis, development, improvement and operation of information and search systems:

PC01 Ability to identify, categorize and formulate software requirements.

PC03 Ability to develop architectures, modules and components of software systems.

PC08 Ability to apply fundamental and interdisciplinary knowledge to successfully solve software engineering tasks.

PC10 The ability to accumulate, process and systematize professional knowledge regarding the creation and maintenance of software and the recognition of the importance of lifelong learning

PC11 Ability to implement phases and iterations of the life cycle of software systems and information technologies based on relevant models and software development approaches.

PC13 The ability to reasonably choose and master software development and maintenance tools.

PC14 Ability to algorithmic and logical thinking.

PC18 Ability to develop methods of numerical solution of mathematical problems using software tools.

The study of the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" contributes to the formation in students of the following **program learning outcomes** (PLO) according to the educational program:

PLO01 To analyze, purposefully search and select the necessary information and reference resources and knowledge to solve professional problems, taking into account modern advances in science and technology.

PLO05 To know and apply relevant mathematical concepts, domain methods, system and objectoriented analysis and mathematical modeling for software development.

PLO06 Ability to select and use the appropriate task of software development methodology.

PLO10 To conduct a pre-project survey of the subject area, system analysis of the design object.

PLO14 To apply in practice instrumental software tools for domain analysis, design, testing, visualization, measurement and documentation of software.

PLO18 To know and be able to apply information technology of processing, storage and transmission of data.

PLO20 To know approaches to evaluation and quality assurance of software.

PLO23 To be able to document and present the software development results.

PLO25 To know and to be able to use fundamental mathematical tools in the algorithms construction and modern software development.

PLO26 To be able to develop and use methods and algorithms for the mathematical problems approximate solution during the multimedia and information retrieval systems design.

2. Prerequisites and post-requisites of the course (the place of the course in the structural-logical scheme of studies in accordance with educational program)

The successful study of the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" is preceded by the study of the disciplines "Theory of Probability", "Mathematical Support of Multimedia and Information Retrieval Systems" of the bachelor's training plan in the specialty 121 Software engineering.

The theoretical knowledge and practical skills obtained during the mastering of the discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" ensure the successful implementation of course and diploma projects in the specialty 121 Software engineering. Also, the acquired knowledge and skills are a prerequisite for successfully mastering the disciplines "Artificial Intelligence Technologies for Information and Retrieval systems", "Modeling and Design of Information Systems" of the master's training plan in the specialty 121 Software engineering.

3. Content of the course

The discipline "Algorithmic Support of Multimedia and Information Retrieval Systems" involves the study of the following topics:

Topic 1. Errors of the result of the numerical solution of the problem

Topic 2. Software methods for solving nonlinear equations

Topic 3. Software methods for solving systems of linear algebraic equations and other linear algebra problems

Topic 4. The selective method and its characteristics. Statistical estimates of distribution parameters

Topic 5. Concept of statistical hypotheses.

Topic 6. Elements of correlation theory.

Modular control work

Credit

4. Educational materials and resources

Basic literature:

1. Onai Mykola. Algorithmic Support of Multimedia and Information Retrieval Systems. Electronic lecture notes. Coogle class.

2. Jeffrey R. Chasnov. (2012). Numerical Methods. The Hong Kong University of Science and Technology. URL: <u>https://www.math.hkust.edu.hk/~machas/numerical-methods.pdf</u>

3. Siegmund, David O. Probability theory. https://www.britannica.com/science/probability-theory. 4. Philip B. Stark. Electronic summary of lectures. Philip B. Stark is Professor of Statistics and Associate Dean of Mathematical and Physical Sciences at the University of California, Berkeley, URL: https://www.stat.berkeley.edu/~stark/SticiGui/Text/reasoning.htm

5. Hartigan, J.A. (1983). Theories of Probability. In: Bayes Theory. Springer Series in Statistics. Springer, New York, NY. https://doi.org/10.1007/978-1-4613-8242-3_1

Educational content

5. Methodology of mastering the discipline (educational component)

Algorithmic support of multimedia and information search systems. Part 1. Numerical methods

No. z/p	Type of training session	Description of the training session
	Topic 1. Errors of the	e result of the numerical solution of the problem
1	<i>Lecture 1.</i> Basic principles of error theory and unstable problems	Classification of errors: task error, method error, computational error. Absolute error. Relative error. Marginal errors. Errors in performing arithmetic operations. Concept and example of unstable problem.
		Task on self-study: analyze types of errors and examples of unstable problems.

2	Practical lesson 1. Elements of the theory of errors	The direct and inverse problem of error theory Task on self-study: to analyze the methods of solving the inverse problem of the theory of errors and the methods of estimating the errors of machine arithmetic.
	Topic 2. Softwa	re methods for solving nonlinear equations
3	Lecture 2. General approaches to solving nonlinear equations, the simplest methods for solving nonlinear equations and types of convergence of iterative sequences	Localization of the roots of a nonlinear equation. Refinement of the roots of a nonlinear equation. Bolzano-Cauchy theorem. Determination of the type and order of convergence of the iterative sequence. Dichotomy methods. Bisection method. Chord method. Newton's method (tangent method). Convergence condition of Newton's method. Criteria for stopping iterative processes for the halving method, the chord method, and the tangent method. Fourier condition.
		Task on self-study: to consider examples of iterative sequences with different order of convergence.
4	Practical lesson 2. Nonlinear scalar equations	Localization of the roots of a nonlinear equation. Dichotomy methods, halving method, chord method and Newton's method (tangent method) for solving nonlinear equations. Task on self-study: to consider examples of iterative sequences with different order of convergence.
5	<i>Laboratory work 1. Nonlinear scalar equations</i>	Nonlinear equations with one unknown (dichotomy methods) Task on self-study: software implementation
6	<i>Lecture 3.</i> The secant method and other modifications of Newton's method	The Newton-Schroder method. Newton's simplified method. Newton's difference method. Stephensen's method. Method of secants and determination of its order of convergence. Hybrid methods of solving nonlinear equations.
		Task on self-study: consider examples of solving nonlinear equations by the secant method and other modifications of Newton's method.
7	Practical lesson 3. Non-linear scalar equations	Newton's simplified method, Newton's difference method, Stephensen's method, the secant method and determining its order of convergence, hybrid methods for solving nonlinear equations
		Task on self-study: consider examples of solving nonlinear equations by the secant method and other modifications of Newton's method.
8	<i>Lecture 4</i> . Method of simple iterations	Fixed point principle. Formulation of the fixed point problem. A priori and a posteriori error estimates for the method of simple iterations. The process of λ -parameterization of a nonlinear equation (reduction of a nonlinear equation to an iterative form).
		Task on self-study: to analyze the process of λ -parameterization for a nonlinear equation with a positive and negative derivative on the interval.
9	Practical exercise 4. Non- linear scalar equations	The method of simple iterations for solving nonlinear equations. Methods of determining the limits of existence of the roots of an algebraic equation. The process of squaring.

10	Laboratory work 2. Nonlinear	Lobachevsky's method of solving algebraic equations. Task on self-study: to analyze the process of λ - parameterization for a nonlinear equation with a positive and negative derivative on the interval. Nonlinear equations with one unknown (Newton's method and
	scalar equations	its modifications) Task on self-study: software implementation
Тој	pic 3. Software methods for solvin	g systems of linear algebraic equations and other linear algebra problems
11	<i>Lecture 5</i> . <i>Method of LU-</i> <i>factorization</i>	Theorem on LU-decomposition. The classical LU decomposition method. Kraut's method. Doolittle's method. Diagonal dominating matrix. Finding the inverse of a matrix using LU decomposition. Calculation of the determinant using the LU- expansion.
		Task on self-study: derive formulas for the LU-factorization of the matrix A when there is a unit diagonal in the left triangular matrix.
12	Practical lesson 5. Software methods of solving SLAR	The classical LU-decomposition method of solving systems of linear algebraic equations
		Task on self-study: derive formulas for the LU-factorization of the matrix A when there is a unit diagonal in the left triangular matrix.
13	<i>Lecture 6. Reflection method</i>	Householder transformation. Reflection matrix. Properties of the Householder matrix. QR matrix decomposition. Finding the determinant using QR decomposition. Application of QR- decomposition in solving SLAR.
		Task on self-study: to find the coordinate formulas for building the matrix (calculation of each component of the matrix), which is obtained after performing the sth stage of the Householder transformation.
14	Practical lesson 6. Software methods of solving SLAR	QR matrix decomposition. Application of QR-decomposition of the matrix in solving SLAR. Task on self-study: to find the coordinate formulas for building the matrix (calculation of each component of the matrix), which is obtained after performing the sth stage of the Householder transformation.
15	<i>Laboratory work 3.</i> Software methods of solving SLAR	LU-factorization method, reflection method Task on self-study: software implementation.
16	Lecture 7. The method of square roots. The method of running	Necessity of constructing a method of solving SLAR with a positive definite matrix. Kholetsky's method. Generalization of the method of square roots for the case of the Hermitian matrix of coefficients. The need to develop methods for solving SLAR with a tape structure. Tridiagonal run. Straight run. Reverse run. Concept of correctness and stability of running. Five-

		diagonal purlin. Left and right run.
		Task on self-study: to derive the formulas of the left sweep for SLAR with a tridiagonal matrix of coefficients.
17	Practical lesson 7. Software methods of solving SLAR	The Kholetskyi method of solving the SLAR with a positive definite matrix
		Task on self-study: to derive the formulas of the left sweep for SLAR with a tridiagonal matrix of coefficients.
18	<i>Lecture 8</i> . Method of rotations and solution of systems of equations with complex coefficients	Growth of the element in the lower right corner of the matrix of coefficients when using the Gaussian method. Construction of a method that would not change the norm of each column vector of the matrix of coefficients. Solving a system of linear algebraic equations with complex coefficients by reducing it to a system of linear algebraic equations with real coefficients.
		Task on self-study: to prove that the Euclidean norm of any column vector of the extended matrix A remains unchanged when the rotation transformations necessary to bring the matrix A to the right triangular form are performed.
19	Practical lesson 8. Software methods of solving SLAR	Method of rotations for solving SLAR Task on self-study: to prove that the Euclidean norm of any column vector of the extended matrix A remains unchanged when the rotation transformations necessary to bring the matrix A to the right triangular form are performed.
20	<i>Laboratory work 4. Software methods of solving SLAR</i>	Method of rotations and solution of systems of equations with complex coefficients Task on self-study: software implementation.
21	Lecture 9.	Modular control work
22	Practical lesson 9.	Results

Algorithmic support of multimedia and information search systems. Part 2. Mathematical statistics

No. z/p	Type of training session	Description of the training session	
То	bic 4. The selective method and in	ts characteristics. Statistical estimates of distribution parameters	
23	<i>Lecture 10.</i> Basics of the selective method of data processing. Calculation of numerical characteristics of the sample.	nod of data population, discrete statistical distribution, polygon and alculation of frequencies and relative frequencies, empirical distribution	
24	Practical lesson 10. Interval evaluation of the sample. Calculation of the numerical characteristics of	Independent work #10. Task on self-study: item 6	

	the sample. Graphical study of the sample.	
25	Lecture 11. Interval statistical distribution of the sample. Methods of finding estimates. Substitution method and Pearson's method. The method of maximum likelihood and the method of least squares.	Graphical methods of representing interval variation series, cumulate, sample mean and variance, median, mode. Types of statistical estimates, methods of finding point estimates, the method of maximum likelihood. Estimation of linear, hyperbolic, parabolic and exponential function parameters. Confidence limits for mathematical expectation, confidence intervals for mathematical expectation, variance and mean squared deviation.
		Task on self-study: item 6
26	Practical lesson 11. Estimation of parameters by methods of substitution,	Independent work #11. Task on self-study: item 6
	Pearson, Fisher and least squares.	
27	Laboratory work 5 .	Selection of software, programming tasks.
	Interval evaluation of the sample. Calculation of numerical characteristics of the sample. Graphical analysis	
	Topic 5	Concept of statistical hypotheses.
28	<i>Lecture 12.</i> Main and alternative hypotheses: verification methods.	Methods of formation of the main and alternative hypotheses, selection of a critical area, methods of testing hypotheses Task on self-study: item 6
29	Practical lesson 12. Main and alternative hypotheses: testing methods.	Independent work #12. Task on self-study: item 6
30	<i>Lecture 13</i> . <i>Tests of hypotheses.</i>	Checking the correctness of the null hypothesis, about distribution parameters, about numerical values. Task on self-study: item 6
31	Practical lesson 13.	
	Checking the correctness of statistical hypotheses	Independent work #13. Task on self-study: item 6
32	Laboratory work 6.	Programming the problem.
	Estimation of parameters by substitution, Pearson, Fisher and least squares methods. characteristics of the sample. Interval estimates. Testing of statistical hypotheses.	
	-	6. Elements of correlation theory.
33	<i>Lecture 14</i> . Correlation <i>dependence, linear correlation.</i>	<i>Functional, statistical, and correlation dependence, linear correlation dependence, regression line, correlation coefficient,</i>

		testing of the hypothesis about the absence of a correlation relationship. Task on self-study: item 6
34	Practical lesson 14. Correlational dependencies. Regression lines.	Independent work #14. Task on self-study: item 6
35	<i>Lecture 15.</i> Nonlinear correlation dependences	Nonlinear correlation dependences, problems Task on self-study: item 6
36	Practical lesson 15. Nonlinear correlation dependences	Independent work #15. Task on self-study: item 6
37	Laboratory work 7 . Correlation dependence, linear correlation.	Programming the problem.
38	<i>Lecture 16. Elements of dispersion analysis</i>	Group, intergroup and total variance, methods of their calculations Task on self-study: item 6
39	Practical lesson 16. Elements of dispersion analysis.	Independent work #16. Task on self-study: item 6
40	<i>Lecture 17</i> . Univariate variance analysis	Dispersion analysis, methods, applications. Task on self-study: item 6
41	Practical lesson 17. Elements of dispersion analysis.	Independent work #17. Task on self-study: item 6
42	Laboratory work 8 . Presentation of research results	Presentation.
43	Lecture 18.	Modular control work
44	Practical lesson 18.	Results

6. Independent work of the student

Discipline " Algorithmic Support of Multimedia and Information Retrieval Systems" is based on independent preparations for classroom classes on theoretical and practical topics.

No. The name of the topic submitted for z/p independent processing		How many hours	literature
	Part 1.	Numeric	al methods
1	Registration for the course. Acquaintance with methodical literature	1	1-5
2	Preparation for lecture 2	1	1-5

3	Preparation for practical class 2	2.5	1-5
4	Preparation for laboratory session 1	2.5	1-5
5	Preparation for the lecture 3	1	1-5
6	Preparation for the lecture 4	1	1-5
7	Preparation for the lecture 5	1	1-5
8	Preparation for practical class 3	2.5	1-5
9	Preparation for the lecture 6	1	1-5
10	Preparation for the lecture 7	1	1-5
11	Preparation for the lecture 8	1	1-5
12	Preparation for practical class 4	2.5	1-5
13	Preparation for laboratory class 4	2.5	1-5
14	Preparation for the lecture 9	1	1-5
15	Preparation for MKR	5	1-5
	Part 2. M	athemat	tical statistics
16	Registration for the course. Acquaintance with methodical literature	1	1-5
17	Preparation of a practical lesson 10	2.5	1-5
18	Preparation for lecture 11	1	1-5
19	Preparation of practical lesson 11	2.5	1-5
20	Preparation for laboratory class 5	2.5	1-5
21	Preparation for lecture 12	1	1-5
22	Preparation for practical class 12	2.5	1-5
23	Preparation for the lecture 1 3	1	1-5
24	Preparation for practical class 13	2.5	1-5
25	Preparation for the laboratory session 6	2.5	1-5
26	Preparation for the lecture 1 4	1	1-5
27	Preparation for practical class 14	2.5	1-5
28	Preparation for the lecture 1 5	1	1-5
29	Preparation of practical lesson 1 5	2.5	1-5
30	Preparation for laboratory class 7	2.5	1-5
31	Preparation for the lecture 1 6	1	1-5
32	Preparation for practical lesson 1 6	2.5	1-5
33	Preparation for lecture 1 7	1	1-5
34	Preparation of practical lesson 1 7	2	1-5
35	Preparation for MKR	5	1-5

Policy and Assessment

3

7. Policy of academic discipline (educational component)

- Attending lectures and practical classes is mandatory.
- Rules of behavior in classes: activity, respect for those present, turning off phones.
- Adherence to the policy of academic integrity.
- Rules for the protection of calculation works : the works must be done according to the option of the student of education, which is determined by his number in the group list.

8. Types of control and rating system of assessment of learning outcomes

Current control: written survey on the topic of the lesson, practical work, laboratory work, calculation work, modular control work.

Calendar control: is conducted twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements.

Semester control: assessment.

Conditions for admission to the semester control: the current rating is more than 30 points.

Algorithmic support of multimedia and information search systems. Part 1. Numerical methods

Points for laboratory works. The number of works is 3. Maximum number of points for laboratory work: 5 points. Evaluation criteria: 5-4 points – the program is complete and correct; 3-1 points – the program has insignificant shortcomings; 0 points - there is no program or the program action is incorrect. The maximum number of points for laboratory works :

5 points × 3 lab.=15 points

Points for modular control work. The modular control paper from the first part of the discipline consists of two papers of 15 points each.

Evaluation criteria for each work:

15-13 points – the decision is correct;

12-9 points – the decision has flaws;

8-3 points – there are significant errors in the decision;

2-0 point – no decision or incorrect decision.

The maximum number of points for the modular control:

15 points × 2 control work = 30 **points**.

Points for calculation work

Calculation work consists of software implementation of SLAR solution methods. The maximum number of points for calculation work is 5.

Evaluation criteria:

5-4 points – all items are completed correctly, the answer is correct, complete, well-argued;

3 points – there are shortcomings or incorrect calculations in some items;

2 points – there are significant errors in the performance of the work;

1-0 points - there is no work or its execution is incorrect.

The maximum number of points for the calculation work: 5 points × 1 = 5 points .

Calculation of the rating scale (R). The rating scale for the discipline is equal to: R 1 = 15 points + 30 points + 5 points – 0 penalty points = 50 points.

Algorithmic support of multimedia and information search systems. Part 2. Mathematical statistics **Points for independent work in practical classes**. Practical classes - 9. The maximum number of points for a practical session: 3 points. Evaluation criteria:

3 points – the answer is complete and correct;

2 points – there are inaccuracies in the answer, but in general the answer is correct;

1-0 points – there is no answer or the answer is incorrect.

The maximum number of points for independent work in practical classes:

3 points × 9 classes. = 27 points

Points for laboratory works.

Maximum number of points for laboratory work: 2 points. Evaluation criteria:

2 points – the program is complete and correct;

1 point – the program has insignificant shortcomings;

0 points - there is no program or the program action is incorrect.

The maximum number of points for laboratory works . Laboratory classes - 4.

2 points × 4 labs = 8 points

Points for the modular test

The modular control work is carried out after the completion of the study of the discipline and is evaluated with 10 points.

Evaluation criteria:

10-8 points – the decision is correct;
7-5 points – the solution has flaws;
4-2 points – there are significant errors in the decision;
1-0 points – no decision or incorrect decision.

The maximum number of points for a modular control :

10 points × 1 control work = 10 **points**.

Points for calculation work

Calculation work consists of 1 task.

Evaluation criteria:

5-4 points – all items are completed correctly, the answer is correct, complete, well-argued;

3-2 points – there are shortcomings or incorrect calculations in some items;

2-0 points - there is no work or its execution is incorrect.

The maximum number of points for the calculation work : 5 points × 1 = 5 **points** .

Calculation of the rating scale (R).

The rating scale for the discipline is equal to:

R₂ = 27 points + 8 points + 10 points + 5 points – 0 penalty points = 50 points.

R= *R*₁ + *R*₂ = 50 points + 50 points = 100 points.

Table of correspondence of rating points to grades on the university scale :

Scores	Rating
100-95	Perfectly
94-85	Very good

84-75	Fine
74-65	Satisfactorily
64-60	Enough
Less than 60	Unsatisfactorily
Admission conditions not met	Not allowed

9. Additional information on the discipline (educational component)

In the case of a semester rating < 30 points, the student has the right to increase the points by writing an additional test paper.

Working program of the academic discipline (syllabus):

Compiled by Ph.D., associate professor M.V. Onai, senior lecturer, V.I. Suschuk-Slyusarenko.

Approved by the PZKS department (protocol No. 13 dated June 22, 2022)

Agreed by the Methodical Commission of the Faculty of Applied Mathematics (protocol No. 9 dated June 24, 2022)

Is designed by Ph.D., Assoc. Prof., Onai M.V., senior lecturer, V.I. Suschuk-Slusarenko.

Adopted by Computer Systems Software Department (protocol № 12 from 26.04.23)

Approved by the Faculty Board of Methodology (protocol № 10 from 26.05.23)

1. Problems of mathematical statistics, general and sample population, discrete statistical distribution, polygon of frequencies and relative frequencies, empirical distribution function

 The initial and central moments of the variation series, the method of calculating the sample mean and sample variance, the method of calculating the asymmetry and excess of the empirical distribution.
 Graphical methods of representing interval variation series, cumulate, sample mean and variance, median, mode.

4. Types of statistical estimates, methods of finding point estimates, the maximum likelihood method.

5. Estimation of linear, hyperbolic, parabolic and exponential function parameters.

6. Confidence limits for mathematical expectation, confidence intervals for mathematical expectation, variance and mean squared deviation.

7. Statistical criterion, hypothesis testing errors, finding critical areas, power of the criterion.

8. Checking the correctness of the null hypothesis, about distribution parameters, about numerical values.

9. Comparison of several dispersions of normal general populations, calculation of theoretical frequencies for binomial, uniform, normal, exponential distributions and Poisson distribution.

10. Pearson consistency criterion, χ^2 -criterion, Kolmogorov consistency criterion.

11. Criterion of signs, Wilcoxon, Mann, Whitney.

12. Functional, statistical, and correlation dependences, linear correlation dependence, regression line, correlation coefficient, hypothesis testing of the absence of a correlation relationship.

13. Nonlinear correlation dependencies, problems

14. Spearman's sample rank correlation coefficient, Kendall's sample rank correlation coefficient.

15. Dispersion analysis, methods, applications.

16. Equal and unequal number of trials at all levels.