MATHEMATICAL SUPPORT OF MULTIMEDIA AND INFORMATION RETRIEVAL SYSTEMS. Part 2. Fundamentals of harmonic analysis and elements of operational calculus

Working program of the academic discipline (Syllabus)

Details of the academic discipline

Level of higher education	*First (undergraduate)*
Branch of knowledge	*12 Information technologies*
Specialty	*121 Software engineering*
Educational program	*Software Engineering of Multimedia and Information Retrieval Systems*
Discipline status	*Normative*
Form of education	*Daytime*
Year of training, semester	*Second year of training, fourth semester*
Scope of the discipline	*Lectures: 36 hours, practical classes: 36 hours, independent work: 63 hours.*
Semester control	*Exam, modular control work, calculation and graphic work, calendar control*
Timetable	*According to the schedule for the spring semester of the current academic year (rozklad.kpi.ua)*
Language of teaching	*Ukrainian*
Information about the course leader / teachers	*Lecturer: doctor of science, professor, Legeza Viktor Petrovych* *legeza@pzks.fpm.kpi.ua* *Practical training: doctor of science, professor, Legeza Viktor Petrovych*
Placement of the course	*fourth semester: https://classroom.google.com/c/MjYyMzg0MDk3NDc5?hl=uk&cjc=cwkyoze*

Program of study discipline:

1. **Description of the educational discipline, its purpose, subject of study and learning outcomes**

Study of the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" allows students to develop the competencies necessary for building mathematical models and algorithms in the process of research and solving practical problems of natural science and information technology.

The purpose of studying the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" is the formation of a higher education student's abilities for abstract thinking, independent analysis and synthesis of complex multidimensional systems, as well as the ability to use the acquired fundamental knowledge at the stages of posing a problem in mathematical and symbolic form with subsequent algorithmization and development of modern software.

The subject of the discipline is "Mathematical support of multimedia and information-search systems. Part 2. Basics of harmonic analysis and elements of operational calculation" are methods, techniques and technologies of mathematical analysis and its additional sections, which make up the theoretical justification and mathematical support of the process of solving a wide range of problems belonging to the field of knowledge 12 "Information technologies".

Study of the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" will contribute to the formation of the following **professional competences** (PC) in students:

PC 08 Ability to apply fundamental and interdisciplinary knowledge to successfully solve software engineering problems.

PC 14 Ability to algorithmic and logical thinking.

PC 15 Ability to apply fundamental and interdisciplinary knowledge to build advanced retrieval algorithms.

PC 16 Ability to develop algorithms for implementing statistical data analysis methods.

PC 18 Ability to develop methods for mathematical problems numerical solutions using software.

PC 20 Ability to apply the acquired fundamental mathematical knowledge to develop calculation methods in the multimedia and information retrieval systems creation.

Study of the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus " will contribute to the formation of the following **program learning outcomes** (PLO) for students under the EP:

PLO 05 To know and apply relevant mathematical concepts, domain methods, system and object-oriented analysis and mathematical modeling for software development.

PLO 07 To know and to apply in practice the fundamental concepts, paradigms and basic principles of the functioning of language, instrumental and computational tools of software engineering.

PLO 25 To know and to be able to use fundamental mathematical tools in the algorithms construction and modern software development.

PLO 26 To be able to develop and use methods and algorithms for the mathematical problems approximate solution during the multimedia and information retrieval systems design.

PLO 27 To be able to use statistical data analysis methods.

PLO 28 To know the mathematical and algorithmic basics of computer graphics and to be able to apply them to develop multimedia software.

2. Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)

Successful study of the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" should be carried out as part of the thoroughly mastered educational material of the disciplines "Mathematical analysis" and "Mathematical support of multimedia and information-search systems. Part 1. Multidimensional integral calculus" of the first and second year of bachelor's training in the specialty 121 Software engineering.

Received during the assimilation of the discipline "Mathematical support of multimedia and information and search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" theoretical knowledge and practical skills are necessary for studying the disciplines "Physical foundations of multimedia systems", "Software support of multimedia systems" of the bachelor's training plan in the specialty 121 Software engineering, as well as the disciplines "Research operations and mathematical programming" and "Information and search systems and services" of the master's training curriculum under the EP "Software engineering of multimedia and information and search systems".

3. Content of the academic discipline.

Discipline (educational component) "Mathematical support of multimedia and information search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" involves the study of the following topics:

Topic 1. Elements of the theory of analytical functions. Differentiation of analytic functions and the Cauchy-Riemann formula. Integration of analytic functions, Cauchy's main theorem and Cauchy's integral formula.

Topic 2. Theory of remainders. Isolated special points of analytical functions. Cauchy's theorem on residues. Application of residues to the calculation of real definite integrals.

Topic 3. Trigonometric Fourier series.

Topic 4. Fourier integral transform.

Topic 5. Laplace transform: definition, conditions of existence, properties, image finding technique.

Topic 6. Laplace transform: basic methods of restoring the original function from its image. The inverse of the Laplace transform. Application of the Laplace transform in practical problems.

Modular control work (MCW).
Examination

4. Educational materials and resources.

Basic literature

1. Narayan Shanti & Mittal P.K. Theory of Functions of a Complex Variable. S. Chand&Co Ltd, 2005. https://www.amazon.com/Theory-Functions-Complex-Variable-Narayan/dp/8121906393#detailBullets_feature_div
2. Edwin L. Woollett. Maxima by Example: Ch.10: Fourier Series, Fourier and Laplace Transforms. September 16, 2010. https://home.csulb.edu/~woollett/mbe10fltrans.pdf
3. Phil Dyke. An Introduction to Laplace Transforms and Fourier Series. School of Computing and Mathematics University of Plymouth (UK). Springer-Verlag London, 2014, 320 p.
4. G.M.Fikhtengol’ts. The Fundamentals of Mathematical Analysis (Int. series of Monographs on pure and applied Mathematics). Volume 2. Elsevier, Pergamon Press. 1965, 518 p.
5. V.A.Ilyin and E.G.Pozyak. Fundamentals of mathematical analysis. Part 2. Mir Publishers, 1982. – 438 p.
6. N.Piskunov. Differential and Integral calculus.Vol. 2, CBS Publishers & Distributors, 2021, 572 p.
7. S.V.Budak, B.M.Fomin. Multiple Integrals, Field Theory and Series**.** An Advanced Course in Higher Mathematics. Mir Publishers; First printing edition, 1973.

Additional literature

8. Y.B.Zel`dovich, A.D.Myshkis. Elements of Applied Mathematics. Mir Publishers, 1976, 656 p.
9. R.Courant. Differential and Integral Calculus. Vol. 2. Ishi press international, 2010, 682 p.
10. G.N.Berman. A problem book in mathematical analysis. MTG Learning Media (P) Ltd., New Delhi/Gurgaon, 2017, 490 p.
11. B.P.Demidovich. Problems in Mathematical Analysis. Gordon & B., 1969, 496 p.
12. V.P. Legeza. Mathematical analysis: a collection of problems. - Kyiv, KPI, Polytechnic Publishing House, 2018. - 240 p.
13. Vladimirov V.M., Puchkov O.A., Shmygevskyi M.V. A collection of problems in higher mathematics (typical calculations). Part 2. - Kyiv: Polytechnic Publishing House, 2003. - 200 p.

5. Methods of mastering an educational discipline (educational component)

№	Type of training session	Description of the training session
Section 1. Elements of the theory of the function of a complex variable. ***Topic 1. Elements of the theory of analytical functions. Differentiation of analytic functions and the Cauchy-Riemann formula. Integration of analytic functions, Cauchy's main theorem and Cauchy's integral formula***		
1	**Lecture №1.** Functions of a complex variable: general concepts and definitions. Limit and continuity of FCV. Properties of continuous FCV in a limited closed region on the complex plane.	Definition of FCV. Curves and regions in the complex plane. The limit of a sequence with complex terms. Cauchy criterion. Boundary and continuity of FCV. Properties of continuous FCV in a limited closed region on the complex plane. Functional and power series with FCV in the complex domain. Uniform convergence. Abel's theorem. Radius of convergence, circle of convergence, area of convergence of power series. Convergence ring. Elementary FCV, their features and differences from functions of a real variable. Tasks for **IWS**: [1], [12], pp. 160-189 1. Calculate: a) $\mathrm{Arcth}\left(\frac{4+3i}{5}\right)$; б). $(-\sqrt{3}+i)^{-6i}$.

		2. Determine the type of curve: a) $z = \mathrm{th}(5t) + \frac{5i}{\mathrm{ch}(5t)}$; б). $z = 3e^{it} - e^{-it}/2$
2	**Practical lesson №1.** The technique of calculating the boundary of the FCV, research on the continuity of the FCV. Peculiarities of differentiation of FCV. Inspection of FCV for analyticity. Cauchy-Riemann conditions.	Actions on elementary FCV. Curves and areas on the plane. The technique of calculating the boundary of the FCV, research on the continuity of the FCZ. Peculiarities of differentiation of FCV. Inspection of FCV for analyticity. Cauchy-Riemann conditions. Tasks for **IWS**: [1], [12], pp. 160-189
3	**Lecture №2.** Differentiation of FCV. Differentiability conditions of FCV. Taylor's series for FCV. The concept of conformal mapping.	Definition of the FCV derivative. Properties of FCV derivatives. Conditions of differentiability of the FCV, its features and differences from the differentiability of the function of a real variable. Cauchy-Riemann conditions. Analytical functions, their properties. Harmonic functions. Differentiation of a power series. Taylor's series for FCV. The geometric content of the derivative. The concept of conformal mapping. Tasks for **IWS**: [1], [12], pp. 160-189 1. Prove that the Cauchy-Riemann conditions take the following form in polar coordinates: $\frac{\partial u}{\partial \rho} = \frac{1}{\rho}\frac{\partial v}{\partial \varphi}$, $\frac{\partial v}{\partial \rho} = -\frac{1}{\rho}\frac{\partial u}{\partial \varphi}$ 2. Find the analytic function $f(z)$ by its imaginary part $v(x,y) = \mathrm{arctg}\left(\frac{y}{x}\right), (x>0)$, if $f(1)=0$.
4	**Practical lesson №2.** Development of FCV in Taylor power series. The technique of integrating FCV in the complex area, peculiarities of FCV integration.	Taylor's power series for FCV. The technique of integrating FCV in the complex area, peculiarities of FCV integration. Tasks for **IWS**: [12], pp. 160-189
5	**Lecture №3.** Integration of FCV. Cauchy's Basic Theorem. Development of an analytic function into a power series.	Definition of the FCV integral and its properties. Integration of functional series. Cauchy's Basic Theorem. Cauchy's formula. Newton-Leibniz formula. Breakdown of an analytic function into a power series. Whole function. Properties of integer functions. Liouville's theorem. Development of elementary functions in the Taylor series. Tasks for **IWS**: [1], [12], pp. 160-189 Calculate the integral: $\int_L e^{\|z\|^2} \operatorname{Im} z^3 dz, if\ L:\{\|z\|=1\}$.
6	**Practical lesson №3.** Mastering the technique of developing the FCV in the Laurent series. Classification of isolated special points. The technique of FCV research at a non-infinitely distant singular point.	Mastering the technique of developing the FCV in the Laurent series. The main and correct parts of the Laurent series. Zero functions. Classification of isolated special points. The technique of FCV research at an infinitely distant special point. Tasks for **IWS**: [12], pp. 160-189
Topic 2. Laurent series. The theory of surpluses. Isolated special points of analytical functions. Cauchy's theorem on residues. Application of remainders to the calculation of real definite integrals.		
7	**Lecture №4.** Series of Laurent. Isolated special points of analytical functions and their classification. An infinitely distant singular point.	Series and Laurent's Theorem. Zero functions. Isolated special points of analytical functions and their classification. The relationship between the nature of isolated singular points of an analytic function and the appearance of its Laurent series. An infinitely distant singular point, its geometric interpretation. The technique of FCV research at an infinitely distant special point. Examples of the development of FCV in the Laurent series. Tasks for **IWS**: [1], [12], pp. 160-189 1. Expand the function $f(z) = \frac{3}{z^2 - 5z + 4}$ in the Laurent series by powers $(z-1)$. 2. For the function $f(z) = \frac{\sin z - z}{(e^{\pi z}+1)(\sin 2z - 2\sin z)}$ find isolated special points and determine their type.
8	**Practical lesson №4.** The concept of surplus. The technique of calculating residues at isolated singular points.	The technique of calculating residues at isolated singular points. Tasks for **IWS**: [1], [12], pp. 160-189
9	**Lecture №5**. Residues. Cauchy's theorem on residues. The technique of calculating remainders in simple and	The definition of residues. Cauchy's theorem on residues. Full plane. An infinitely distant point. Surplus at point . The technique of calculating remainders. Calculation of residuals in a removable singular point, in simple and multiple poles, in a significant singular point.

	multiple poles, at a significantly special point .	Tasks for **IWS**: [1], [12], pp. 160-189 1. Find the residues of the given function $f(z) = z^2 \operatorname{ch}\left(\frac{z-1}{z-2}\right)$ at the point $z_0 = 2$ and then calculate the integral $\oint_L f(z)dz$, $L: \lvert z-2 \rvert = \varepsilon, \varepsilon > 0$. 2. Find the residues of the given function $f(z) = \frac{\sin(z) - z}{z^5(z^2+4)^2}$ in all poles and then calculate the integral інтеграл $\oint_L f(z)dz$, $L: \lvert z \rvert = e$.
10	**Practical lesson №5. On the first half-pair**: Application of remainders to the calculation of real definite integrals (proper and improper). **In the second half**: MCW on topics №№1-2.	Application of remainders to the calculation of real definite integrals (proper and improper). Tasks for **IWS**: [1], [12], pp. 160-189
11	**Lecture №6.** The technique of applying remainders to the calculation of real integrals.	Application of remainders to the calculation of real integrals. Different types of integrals of a real variable, which are calculated on the basis of the transition to the FCW. Fresnel integrals. Logarithmic excess. Tasks for **IWS**: [1], [12], pp. 160-189 1. Calculate the real integral: $\int_0^{2\pi} \frac{dx}{(\sqrt{7}+\sqrt{5}\cos x)^2}$. 2. Calculate the improper integral: $\int_{-\infty}^{\infty} \frac{(x^2+1)dx}{(x^2+2x+2)^2}$.
12	**Practical lesson №6.** The technique of developing $2\pi-$periodic functions into Fourier series. The technology of using Fourier series to calculate the sums of some convergent numerical series.	The technique of developing $2\pi-$periodic functions into Fourier series. Application of Fourier series to the calculation of sums of some convergent numerical series. Tasks for **IWS**: [2, 3], [12], pp. 190-204
	Section II. Mathematical foundations of harmonic analysis. ***Topic 3. Trigonometric Fourier series.***	
13	**Lecture №7**. Periodic functions: general concepts and definitions. Harmonic oscillations, amplitude, frequency, period and phase of oscillations Definition of trigonometric series.	Periodic and aperiodic functions. Harmonic oscillations of a material point. The main characteristics of the oscillatory process: amplitude, frequency, period and phase of oscillations. A class of piecewise smooth functions. Concept and convergence of trigonometric series. Conditions of uniform convergence of trigonometric series. to its sum - functions $f(x)$ on the segment $[-l,l]$. Definition and orthogonality of the basic trigonometric system of functions on the segment $[-l,l]$. Task for **IWS**: [2, 3], [12], pp. 190-204 1. Determine how many times the given trigonometric series can be differentiated term by term : a). $\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^{3,5}}$; б). $\sum_{n=1}^{\infty} \frac{\sin(nx)}{n^{4,01}}$. How many continuous derivatives do the sums of these series have? 2. Prove the orthogonality of the basic trigonometric system of functions on the segment $[-l,l]$.
14	**Practical lesson №7.** Peculiarities of constructing Fourier series for even and odd functions with period $2\pi(2l)$. Fourier series for functions given on an arbitrary segment [a, b].	The technique and peculiarities of constructing Fourier series for even and odd functions with period $2\pi(2l)$. Fourier series for functions given on an arbitrary segment [a , b] . Tasks for **IWS**: [2,3], [12], pp. 160-189
15	**Lecture №8**. Fourier series for $2\pi-$periodic functions. Riemann's lemma for the segment. Theorem on the existence of a Fourier series for a function $f(x)$ at its differentiability points. Integral, kernel and Dirichlet conditions.	Fourier series for $2\pi-$periodic functions. Calculation of the coefficients of the trigonometric Fourier series. Riemann's lemma for the segment. Theorem on the existence of a Fourier series for a function $f(x)$ at its differentiability points. Integral, kernel and Dirichlet conditions. Lipshitz conditions. Tasks for **IWS**: [2,3], [12], pp. 190-204 1. Represent the given function by Fourier series

		$f(x)=\begin{cases}2x+4, if\ -\pi<x<0;\\ 3x-5, if\ 0\le x<\pi.\end{cases}$, periodically extended to the entire numerical axis with a period of 2π. 2. On what grounds (conditions) are the coefficients of the Fourier series derived? 3. What are the Dirichlet conditions? 4. What type of convergence is observed at the points of continuity of the function $f(x)$? 5. What is the value of the sum of the Fourier series at the discontinuity points of the function $f(x)$?
16	**Practical lesson №8.** The technique of constructing Fourier series from sines and cosines for functions given on the segment (on a half-period) [0,*l*]. Application of Fourier series to the calculation of sums of some convergent numerical series.	The technique of constructing Fourier series from sines and cosines for functions given on the segment (on a half-period) [0, *l*] . Application of Fourier series to the calculation of the sums of some convergent numerical series. Task for **IWS**: [2, 3], [12], pp. 190-204
17	**Lecture №9**. The complex form of the Fourier series for functions with period $2\pi\ (2l)$. Fourier series for even and odd functions with period $2\pi\ (2l)$.	Fourier series for functions with period $2\pi\ (2l)$ in complex form. Fourier series for even and odd functions with period $2\pi\ (2l)$. Harmonics, complex amplitudes of harmonics, wave numbers of functions, discrete spectrum. Fourier series for periodic functions given on an arbitrary segment $[a,b]$. Fourier series of cosines for functions given on the segment $[a,b]$. Fourier series of sines for functions given on a segment $[a,b]$. Application of trigonometric series to calculation of sums of some convergent numerical series. Tasks for **IWS**: [2, 3], [12], pp. 190-204 1. Write the Fourier series in complex form for the periodic function $F(x)$, formed by the periodic continuation of the function Фур'є $f(x)=\begin{cases}0,\ -1<x<0;\\ 1,\ 0\le x\le 1.\end{cases}$ on the entire numerical axis with the period $2l=2$. 2. Expand the function $f(x)=x^2$ given on the segment into a Fourier series [3,5], $f(x+2)=f(x)$, $x\in(-\infty;+\infty)$. Using this development, find the sum S of a number series $S=\sum\limits_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$. 3. Expand into a Fourier series the function $f(x)=x-\frac{x^2}{2}$, given on a half-period $[0,2]$ and extended in an odd way with the period $T=4$ on the entire numerical axis. Find the sum of a number series: $S=\sum\limits_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^3}$.
18	**Practical lesson №9.** The technique of constructing Fourier series in complex form for functions with period $2\pi(2l)$	Техніка побудови рядів Фур'є в комплексній формі для функцій з періодом $2\pi(2l)$. Tasks for **IWS**: [2, 3], [12], pp. 190-204
19	**Lecture №10**. Elements of harmonic analysis. Evaluation of the residual of the Fourier series. Gibbs phenomenon.	Elements of harmonic analysis. Evaluation of the residual of the Fourier series. Gibbs phenomenon. Summary: different options for writing the Fourier series. Tasks for **IWS**: [2, 3], [12], pp. 190-204 1. What is the Gibbs phenomenon? 2. Give different options for writing the Fourier series. 3. What are called the amplitude and phase spectra of the function $f(x)$?
20	**Practical lesson № 10.** The real form of the Fourier integral. Repeated Fourier integral. Calculation of amplitude-frequency and phase-frequency characteristics of a given function.	The real form of the Fourier integral. Repeated Fourier integral. Calculation of amplitude-frequency and phase-frequency characteristics of a given function. Tasks for **IWS**: [2, 3], [12], pp. 190-204

Topic 4. Fourier integral transform.		
21	**Lecture №11.** The real Fourier integral as a continuous analogue of the Fourier series. Riemann's lemma for an infinite interval $(-\infty,+\infty)$. Sufficient conditions for the function to be represented by a real Fourier integral. The complex form of the Fourier integral.	Aperiodic functions and concepts of the Fourier integral. Unlimited expansion of the interval $(-l;l)$ of the development of the function $f(x)$ into the Fourier series. Sufficient conditions for the representation of the function $f(x)$ by the Fourier integral. Justification of the transition from the Fourier series to the Fourier integral. The complex form of the Fourier integral. Double Fourier integral in complex form. Tasks for **IWS**: [2, 3], [12], pp. 190-204 Use the Fourier integral to represent the following functions: 1. $f(x)=\begin{cases}0, \lvert x\rvert>1;\\ 1, 0\le x\le 1;\\ -1, -1\le x<0.\end{cases}$ and calculate $\int\limits_0^\infty \frac{\sin^3 t}{t}dt$; 2. $f(x)=\begin{cases}\sin x, \lvert x\rvert\le\pi;\\ 0, \lvert x\rvert>\pi.\end{cases}$
22	**Practical lesson №11.** Peculiarities of representing even and odd functions in the form of a Fourier integral. Complex form of Fourier integrals.	Peculiarities of representing even and odd functions in the form of a Fourier integral. Complex form of Fourier integrals. Tasks for **IWS**: [2, 3], [12], pp. 190-204
23	**Lecture №12.** Direct and inverse Fourier transform. Fourier image. Spectral (frequency) characteristics of signals. Sine and cosine Fourier transforms. Integral equations.	Fourier transformation as a mathematical basis for solving the physical problem of efficient conversion of analog signals into digital ones. Definition of the direct and inverse Fourier transform. Amplitude-frequency and phase-frequency characteristics of the signal. Theorem on the conditions for the existence of the direct and inverse Fourier transform. The formula for restoring the "original function" by a given Fourier transform (image). Fourier integral for even and odd functions. Double Fourier integral. Sine and cosine Fourier transforms. Law of reciprocity of sine and cosine Fourier transforms. Integral equations. Discontinuous Dirichlet multiplier. Tasks for **IWS**: [2, 3], [12], pp. 190-204 Find the Fourier transform of the following functions: 1. $f(x)=\frac{\sin x}{1+x^4}$; 2. $f(x)=\frac{x}{(1+x^2)^2}$; 3. $f(x)=\frac{x\cdot\cos x}{(1+x^2)^2}$; 4. $f(x)=\frac{d}{dx}\left(x\cdot e^{-\lvert x\rvert}\right)$; 5. $f(x)=\frac{d^2}{dx^2}\left(x\cdot e^{-\lvert x\rvert}\right)$.
24	**Practical lesson №12.** Direct and inverse Fourier transform. Spectral density and its modulus for a given function.	The technique of using the direct and inverse Fourier transform. Determination of spectral density and its modulus for a given function. Tasks for **IWS**: [2, 3], [12], pp. 190-204
25	**Lecture №13**. Fourier series for orthogonal systems of functions. Bessel's inequality. Concept of completeness and closure of orthogonal systems of functions. Parseval's equality.	The concept of uniform approximation of functions. Basic Theorems on the uniform ε–approximation of a continuous function by trigonometric and algebraic polynomials. Theorem on the conditions of uniform convergence of the Fourier series. Weierstrass' First Theorem on uniform ε–approximation of a continuous function by trigonometric polynomials. Second Weierstrass Theorem on the uniform ε–approximation of a continuous function by algebraic polynomials. Orthogonal systems of functions. Fourier coefficients and Fourier series for a function $f(x)$ according to an orthogonal system of functions. The least square deviation problem. Bessel's identity. Bessel's inequality. Completeness and closure of orthogonal systems of functions. Parseval's equality. Tasks for **IWS**: [2, 3], [12], pp. 190-204 Prove the theorem on the completeness of the basic trigonometric system.
26	**Practical lesson №13. In the first half-pair:** Techniques of sine and cosine Fourier transforms. Solving integral equations using sine and cosine transformations Фур'є. **In the second half**: MCW on topics No. 3-4	Fourier sine and cosine transform technique. Solving integral equations using sine and cosine transformations. Tasks for **IWS**: [2, 3], [12], pp. 190-204

Section III. Elements of operational calculus. ***Topic 5. Laplace transform: definition, conditions of existence, properties, image finding technique.***		
27	**Lecture №14.** Laplace transform: general concepts and definitions. Original and image. Theorem on the conditions for the existence of an image. Analytical image.	Laplace transform: general concepts and definitions. Original and image. Checking the specified functions for "originality". Theorem on the unity of the image. Theorem on the conditions for the existence of an image. Analytical image. Theorem on the necessary sign of the existence of an image. The technique of finding images for original functions by definition. Heaviside function. Tasks for **IWS**: [2, 3], [12], pp. 205-234. Find the image of the given function by definition: а) $f(t)=\cos^5(t)$; б) $f(t)=\cos(2t)\cdot\cos(4t)\cdot\mathrm{sh}(t)\cdot\mathrm{sh}(3t)$; в). $f(t)=\dfrac{e^t-1}{t}$; г). $f(t)=\dfrac{\sin(\omega\cdot t)}{t}$; д). $f(t)=\dfrac{\cos\alpha t-\cos\beta t}{t}$; е). $f(t)=\dfrac{\mathrm{sh}(t)-\sin(t)}{t^2}$. 2. Prove the analyticity of the image $F(p)$. 3. Prove the necessary sign of the existence of the image.
28	**Practical lesson No. 14.** Laplace transform. Finding images by definition. Installing images for elementary functions**.**	Laplace transform. Finding images by definition. Installing images for elementary functions. Tasks for **IWS**: [2, 3], [12], pp. 205-234.
29	**Lecture №15.** Properties of the Laplace transform. Table "original-image". The technique of finding images for original functions by the properties of the Laplace transform.	Properties of the Laplace transform. Table "original-image". The technique of finding images for original functions by the properties of the Laplace transform. Image of a periodic signal. Corollaries about the limit relations arising from the Theorem on the differentiation of the original. Multiplication of images. Roll up of originals. Duhamel's formula. Reproduction of originals. Collapsing images. Tasks for **IWS**: [2, 3], [12], pp. 205-234. 1. Prove the formula for representing a periodic signal. 2. Prove the convolution image theorem. 3. Prove the consequences of the limit relations arising from the theorem on the differentiation of the original.
30	**Practical lesson No.15.** Using the properties of the Laplace transform to find images: attenuation, delay, advance, differentiation by parameter.	Using the properties of the Laplace transform to find images: attenuation, delay, advance, differentiation by parameter. Tasks for **IWS**: [2, 3], [12], pp. 205-234.
Topic 6. Laplace transform: basic methods of restoring the original function from its image. The inverse of the Laplace transform. Application of the Laplace transform in practical problems.		
31	**Lecture №16.** Techniques and methods of finding originals. A generalized development theorem on the inverse of the Laplace transform.	Methods of finding originals: the method of decomposing a rational fraction into the sum of simpler ones; methods based on three development theorems. A generalized development theorem on the inverse of the Laplace transform. Application of normal and double convolution for finding originals. Sufficient conditions under which the analytic function $F(p)$ will be the image of a certain original function $f(t)$. Tasks for **IWS**: [2, 3], [12], pp. 205-234. Find the originals of given images: 1. $F(p)=\dfrac{5(p-9)}{p(p-1)(p^2+9)}$; 2. $F(p)=\dfrac{1}{p}e^{1/p}$; 3. $F(p)=\dfrac{1}{(p^2+1)^3}$ 4. $F(p)=\dfrac{p^2}{(p^2+1)^3}$. 5. $F(p)=\ln\left(\dfrac{p}{p-1}\right)$; 6. $F(p)=\dfrac{1}{2}\ln\left(\dfrac{p^2+1}{p^2+4}\right)$.
32	**Practical lesson №16.** Using the properties of the Laplace transform: differentiation and integration of the original; image differentiation and integration, image multiplication. The	Using the properties of the Laplace transform: differentiation and integration of the original; image differentiation and integration, image multiplication. The concept of convolution of functions and the use of Duhamel's formula. Tasks for **IWS**: [2, 3], [12], pp. 205-234.

	concept of convolution of functions and the use of Duhamel's formula	
33	**Lecture №17.** Application of operational calculus to solving differential equations and their systems. Peculiarities of applying Duhamel's formula to the solution of LDE.	A general approach to solving linear differential equations (LDEs) and their systems with constant coefficients. Peculiarities of solving LDEs according to Duhamel's formula Tasks for **IWS**: [2, 3], [12], pp. 205-234. 1. Is it possible to use Duhamel's formula with non-zero initial conditions? If so, how should it be done? 2. Solve the linear LDE: $y''+2y'+y=\frac{te^{-t}}{1+t}$; $y(0)=y'(0)=0$. 2. Solve the Volterra equation: $y(t)=\eta(t)+e^{2t}+\frac{1}{6}\int_0^t(t-\tau)^3y(\tau)d\tau$. 3. Solve the linear LDE: $x''+x=f(t)$; $x(0)=x'(0)=0$; where $f(t)=\begin{cases}\cos t,\ 0\le t<\pi;\\ 0,\ t\ge\pi.\end{cases}$
34	**Practical lesson №17.** Methods of restoring the original function from a given image function. The method of decomposing a rational fraction into a sum of elementary fractions. The method of finding the original using the three main Development Theorems. Transition to FCV and theory of residuals.	Methods of restoring the original function from a given image function. The method of decomposing a rational fraction into a sum of elementary fractions. The method of finding the original using the three main Development Theorems. Transition to FCV and theory of residuals. Tasks for **IWS**: [2, 3], [12], pp. 205-234.
35	**Lecture №18.** Application of operational calculus to solving integral Volterra equations. Solving ordinary differential equations with a graphically specified right-hand side.	Application of operational calculus methods to solving integral Volterra equations. Solving ordinary differential equations with a graphically specified right-hand side. Tasks for **IWS**: [2, 3], [12], pp. 205-234.
36	**Practical lesson №18. On the first half-pair:** Application of the Laplace transform to the solution of systems of linear differential and integral equations of the Volterra type. Features of the differential equations solution with a graphical right part**. In the second half:** MCW on topics No. 5-6	Application of the Laplace transform to the solution of systems of linear differential and integral equations of the Volterra type. Features of the differential equations solution with a graphical right part. Tasks for **IWS**: [2, 3], [12], pp. 205-234.

6. Independent work of the student (IWS)

Mastering the educational material from the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" is based on self-preparation for classroom classes on theoretical and practical topics.

№	*The name of the topic submitted for independent processing*	*Number of hours*	*Literature*
1	*Preparation for the lecture 1.*	*0.5*	*1, lecture №1, 12, p.p. 160 - 189, 13, p.p..63 – 97.*
2	*Preparation for practical lesson 1*	*0.5*	*12, p.p. 160 - 189, 13, p.p..63 – 97.*
3	*Preparation for the lecture 2*	*0.5*	*1, lecture №2, 12, p.p. 160 - 189, 13, p.p..63 – 97.*
4	*Preparation for practical lesson 2*	*0.5*	*12, p.p. 160 - 189, 13, p.p..63 – 97.*

5	*Preparation for the lecture 3*	*0.5*	*1, lecture №3, 12, p.p. 160 - 189, 13, p.p..63 – 97.*
6	*Preparation for practical lesson 3*	*0.5*	*12, p.p. 160 - 189, 13, p.p..63 – 97.*
7	*Preparation for the lecture 4*	*0.5*	*1, lecture №4, 12, p.p. 160 - 189, 13, p.p..63 – 97.*
8	*Preparation for practical lesson 4*	*0.5*	*12, p.p. 160 - 189, 13, p.p..63 – 97.*
9	*Preparation for the lecture 5*	*0.5*	*1, lecture №5, 12, p.p. 160 - 189, 13, p.p..63 – 97.*
10	*Preparation for practical lesson 5*	*0.5*	*12, p.p. 160 - 189, 13, p.p..63 – 97.*
11	*Preparation for the lecture 6*	*0.5*	*1, lecture №6, 12, p.p. 160 - 189, 13, p.p..63 – 97.*
12	*Preparation for practical lesson 6*	*0.5*	*12, p.p. 190 -204, 13, p.p. 21-34.*
13	*Preparation for the lecture 7*	*0.5*	*2,3, lecture №7, 12, p.p. 190 -204, 13, p.p. 21-34..*
14	*Підготовка до практичного заняття 7*	*0.5*	*12, p.p. 190 -204, 13, p.p. 21-34.*
15	*Preparation for the lecture 8*	*0.5*	*2,3, lecture №8, 12, p.p. 190 -204, 13, p.p. 21-34.*
16	*Preparation for practical lesson 8*	*0.5*	*12, p.p. 190 -204, 13, p.p. 21-34.*
17	*Preparation for the lecture 9*	*0.5*	*2,3, lecture №9, 12, p.p. 190 -204, 13, p.p. 21-34.*
18	*Preparation for practical lesson 9*	*0.5*	*12, p.p. 190 - 204, 13, p.p. 21-34.*
19	*Preparation for the lecture 10*	*0.5*	*2,3, lecture №10, 12, p.p. 190 - 204, 13, p.p. 21-34.*
20	*Preparation for practical lesson 10*	*0.5*	*12, p.p. 190 - 204, 13, p.p. 21-34.*
21	*Preparation for the lecture 11*	*0.5*	*2,3, lecture №11, 12, p.p. 190 - 204, 13, p.p. 21-34.*
22	*Preparation for practical lesson 11*	*0.5*	*12, p.p. 190 - 204, 13, p.p. 21-34.*
23	*Preparation for the lecture 12*	*0.5*	*2,3, lecture №12, 12, p.p. 190 - 204, 13, p.p. 21-34.*
24	*Preparation for practical lesson 12*	*0.5*	*12, p.p. 190 - 204, 13, p.p. 21-34.*
25	*Preparation for the lecture 13*	*0.5*	*2,3, lecture №13, 12, p.p. 190 - 204, 13, p.p. 21-34.*

26	*Preparation for practical lesson 13*	*0.5*	*12, p.p. 190 - 204, 13, p.p. 21-34.*
27	*Preparation for the lecture 14*	*0.5*	*2,3, lecture №14, 12, p.p. 205 - 234, 13, p.p. .99 – 132.*
28	*Preparation for practical lesson 14*	*0.5*	*12, p.p. 205 - 234, 13, p.p. .99 – 132.*
29	*Preparation for the lecture 15*	*0.5*	*2,3, lecture №15, 12, p.p. 205 - 234, 13, p.p. .99 – 132.*
30	*Preparation for practical lesson 15*	*0.5*	*12, p.p. 205 - 234, 13, p.p..99 – 132.*
31	*Preparation for the lecture 16*	*0.5*	*2,3, lecture №16, 12, p.p. 205 - 234, 13, p.p..99 – 132.*
32	*Preparation for practical lesson 16*	*0.5*	*12, p.p. 205 - 234, 13, p.p..99 – 132.*
33	*Preparation for the lecture 17*	*0.5*	*2,3, lecture №17, 12, p.p. 205 - 234, 13, p.p..99 – 132.*
34	*Preparation for practical lesson 17*	*0.5*	*12, p.p. 205 - 234, 13, p.p..99 – 132.*
35	*Preparation for the lecture 18*	*0.5*	*2,3, lecture №18, 12, p.p. 205 - 234, 13, p.p..99 – 132.*
36	*Preparation for practical lesson 18*	*0.5*	*12, p.p. 205 - 234, 13, p.p..99 – 132.*
37	*Preparation for MCW*	*5*	*1-3, 6, 7, 11-13.*
38	*Preparation for IWS*	*10*	*12, c. 160–234.*
39	*Preparation for the exam*	*30*	*1-3, 6,7, 11-13*

Policy and control

7. Policy of academic discipline (educational component)

1. **General policy of teaching** the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" is aimed at independent performance of educational tasks, tasks of current and final control of learning results (for persons with special educational needs, this requirement is applied taking into account their individual needs and capabilities); mandatory correct reference to sources of information in the case of using other people's ideas, developments, statements, technologies; providing reliable information about the results of one's own educational (scientific, creative) activities, used research methods, technologies and sources of information,
2. **Visitation Policy**. In the normal course of study, attendance at both lectures and practical classes is a mandatory component of assessment. In long-term force majeure circumstances (military actions, pandemics, international internships), training can be conducted remotely. In this case, the absence of a classroom lesson does not involve the calculation of penalty points, since the student's final rating score is formed exclusively during the final examination. At the same time, independent performance of modular control tasks and defense of individual thematic tasks, as well as speeches (reports) at colloquiums and active work in practical classes will be evaluated during classroom classes.
3. **Policy on working out and redoing assessment control measures**. According to the regulation "Regulations on current, calendar and semester control of study results at Igor Sikorsky Kyiv

Polytechnic Institute" (https://kpi.ua/files/n3277.pdf) every student has the right to make up for classes missed for a good reason and assessment control measures (hospital, mobility, etc.) at the expense of independent work.

4. **The procedure for contesting the results of assessment control measures**. According to the "Regulations on the resolution of conflict situations in the Igor Sikorsky Kyiv Polytechnic Institute" (https://osvita.kpi.ua/node/169) students have the right to challenge the results of the control measures with arguments, explaining which criterion they disagree with according to the assessment. A student may raise any issue relating to the assessment procedure and expect it to be dealt with in accordance with pre-defined procedures.
5. **Academic integrity**. The policy and principles of academic integrity are governed by the norms set forth in Chapter 3 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).
6. **Norms of ethical behavior**. The norms of ethical behavior of students and scientific and pedagogical workers are regulated by the provisions set forth in Chapter 2 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).
7. **Inclusive education**. Acquisition of knowledge and skills in the course of studying the discipline "Mathematical support of multimedia and information-search systems. Part 2: Fundamentals of harmonic analysis and elements of operational calculus" is accessible to most people with special educational needs, except for those with severe visual impairments who cannot complete tasks using personal computers, laptops and/or other technical aids.
8. **Calendar control** is carried out in order to improve the quality of students' education and monitor the student's fulfillment of the syllabus requirements. Read more: Chapter 3 "Regulations on current, calendar and semester control of study results at the Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf).
9. **Studying in a foreign language**. In the process of mastering the lecture material and performing practical tasks in the discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" students are recommended to refer to the English-language sources listed in the lists of basic and additional literature.
10. **Assignment of incentive and penalty points**. In accordance with the "Regulations on the system of evaluation of learning results at the Igor Sikorsky Kyiv Polytechnic Institute" the sum of all incentive points cannot exceed 10% of the rating scale (https://osvita.kpi.ua/node/37). The rules for assigning incentive and penalty points are as follows.

    Incentive points are awarded for: a) writing theses, articles, design of a new mathematical problem/technology as a scientific work for participation in a competition of student scientific works (on the subject of the academic discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus") – up to 2 points; b) participation in international or all-Ukrainian events and competitions (on the subject of the academic discipline "Mathematical support of multimedia and information-search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus"*)* - up to 3 points.

    Penalty points are awarded for violations of the principles of academic integrity (non-independent performance of MCW and IWS, writing off during the exam): - 5 points for each violation (attempted plagiarism).

    Self-examination, preparation for practical classes, performance of individual tasks and control measures are carried out during independent work of students with the possibility of consulting with the teacher at the specified consultation time or by means of electronic correspondence (e-mail, messengers).

8. Types of control and rating system for evaluating learning outcomes (ELO)

Rating of the discipline "Mathematical support of multimedia and information and search systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus" consists of:
1) points for individual calculation and graphic work (IWS),

2) points for the integrated modular control work (MCW),
3) points for answering the exam,
4) incentive points,
5) penalty points.

RATING SYSTEM OF EVALUATION (ELO)

8.1 Points for the implementation and protection of an individual IWS.

During each semester, students perform one individual IWS, which is divided into all thematic sections.

The maximum number of points for a semester individual IWS: 20 points (total).

Points are awarded for:
- quality of execution (for individual IWS): 0-8 points;
- answer during the defense (for individual IWS): 0-8 points;
- timely submission of work for defense: 0-4 points.

Performance evaluation criteria:
8 points – the work is done qualitatively, in full;
6 points - the work is done qualitatively, in full, but has shortcomings;
3 points - the work is completed in full, but contains minor errors;
0 points – the work is incomplete or contains significant errors.
Criteria for evaluating the quality of the answer:
8 points – the answer is complete, well-argued;
6 points – the answer is generally correct, but has flaws or minor errors;
3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Criteria for evaluating the timeliness of work submission for defense:
4 points – the work is presented for defense no later than the specified deadline;
2 points – the work is submitted for defense 1 week later than the specified deadline;
0 points – the work is submitted for defense more than 1 week later than the specified deadline.
The maximum number of points for the implementation and defense of the semester individual IWS: 8+8+4=20 points (in total for three thematic sections).

8.2 Points for the completion of the semester integrated modular control work (MCW).

During the semester, students complete one semester-long integrated modular test, divided by thematic sections into four equal parts in terms of points and time; all tasks are written, including one theoretical and three practical.

The maximum total number of points for the integrated semester MCW is 30 points.

Criteria for evaluating written tasks of the integrated MCW:
30 points - the solution of the MCW tasks is absolutely (100%) correct;
27 points - the solution of the vast majority of tasks is correct, but in 10% of the tasks there are insignificant errors;
24 points – most tasks are solved correctly, but 20% of the tasks contain errors;
10-15 points - half of the tasks are solved correctly, but 50% of the tasks have significant errors;
5-6 points - the solution of 20% of the tasks is correct, but there are significant errors in 80% of the tasks;
1-3 points – the solution of 10% of tasks is correct, but 90% of tasks have significant errors;
0 points - there is no answer or the answer is 100% incorrect.

The semester component of the rating scale**: $R_S = R_{IWS} + R_{MCW}$ = 20+30 points = 50 points**.

8.3. Penalty points.

Penalty points are calculated for:
- academic dishonesty (plagiarism, non-independent performance of MCW, IWS, etc.) - 5 points for one attempt.

8.4. Points for answers on the exam.

The examination ticket consists of 6 questions - 1 theoretical and 5 practical. The answer to a theoretical question is worth 10 points, and the answer to each practical question is worth 8 points.
Evaluation criteria for the theoretical question of the examination paper:
10 points – the answer is correct, complete, well-argued;
8-9 points – the answer is correct, detailed, but not very well argued;
6-7 points - in general, the answer is correct, but has flaws;
4-5 points – there are minor errors in the answer;
1-3 points – there are significant errors in the answer;

0 points - there is no answer or the answer is incorrect.
Evaluation criteria for the practical question of the examination work:
8 points – the answer is correct, the calculations are completed in full;
6-7 points - the answer is correct, but not very well supported by calculations;
5 points - in general, the answer is correct, but has flaws;
3-4 points – there are minor errors in the answer;
1-2 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.

The maximum number of points for an answer on the exam:
R_E = 10 points × 1 theoretical question + 8 points × 5 practical tasks = 50 points.

8.5. Calculation of the rating scale (R).
The semester component of the rating scale R_S= 50 points, it is defined as the sum of positive points received for the completion of the integral modular control work, for the completion and defense of the individual IWS and negative penalty points.
The examination component of the rating scale is equal to: R_E = 50 points.
The rating scale for the discipline is equal to: R = R_S + R_E = 100 points.

8.6. Calendar control: is carried out twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements:
At the first certification (8th week), the student receives "credited" if his current rating is at least 10 points (50% of the maximum number of points a student can receive before the first certification).
At the second certification (14th week), the student receives "passed" if his current rating is at least 20 points (50% of the maximum number of points a student can receive before the second certification).
8.7. Conditions for admission to the exam and determining the grade.
A necessary condition for a student's admission to the exam is the completion and defense of all individual works and the student's semester rating of at least 60% of the R_S, i.e. at least 30 points. Otherwise, the student must do additional work and improve his rating.
The total rating of the R student is defined as the sum of the semester rating of the student R_S and the R_E points received on the exam. The grade is assigned according to the value of R according to the table. 1.

Table 1

Total rating R_D	**Grade**
95-100	excellent
85-94	very good
75-84	good
65-74	satisfactory
60-64	enough
$\mathbf{R_D} \le 59$	unsatisfactory
$\mathbf{r_C}$ < 30 or not performed (not protected) all types of work.	not allowed

8.8 Additional information on the discipline (educational component)
A copy of the typical exam tickets issued for each semester control is given in Appendix 1.

Working program of the academic discipline (syllabus):
Compiled by DcS., prof. Legeza V.P.
Adopted by Computer Systems Software Department (protocol № 12 from 26.04.23)

Approved by the Faculty Board of Methodology (protocol № 10 from 26.05.23)

Appendix 1

TYPICAL EXAMINATION TICKET FROM THE DISCIPLINE "MATHEMATICAL SUPPORT OF MULTIMEDIA AND INFORMATION SEARCH SYSTEMS- Part 2. Fundamentals of harmonic analysis and elements of operational calculus" FOR SEMESTER CONTROL (FOURTH SEMESTER)

<table>
<tr><td colspan="4" align="center">НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ УКРАЇНИ «КПІ» ім. І.Сікорського</td></tr>
<tr>
<td>Educational and qualification level Bachelor Specialty 121 "Software Engineering"</td>
<td>Department of software of computer systems

2022 – 2023 education year</td>
<td>EXAMINATION TICKET No. 10 from the academic discipline

MATHEMATICAL SUPPORT OF MULTIMEDIA AND INFORMATION SEARCH SYSTEMS-2
4-th semester</td>
<td>I approve
Chief Department of software of computer systems
____
(signature)
DcS., Assoc.prof.
E.S. Sulema

Prot. No. 13 dated 06/22/2022</td>
</tr>
<tr><td colspan="4" align="center">Examination theoretical questions</td></tr>
<tr><td colspan="4">1. Theorem on the conditions imposed on the original function to ensure the existence and analyticity of its image (with proof). In your opinion, what are these conditions related to from a practical point of view?</td></tr>
<tr><td colspan="4" align="center">Practical tasks of various types</td></tr>
<tr><td colspan="4">2. Establish the type of singular points and find the residuals of the function relative to them: $f(z)=\dfrac{1}{(z-i)^3(z^2+4)}$. Calculate the integral $\int\limits_{|z|=\pi} f(z)dz$.</td></tr>
<tr><td colspan="4">3. The function $f(x)=2(x-22)/3$ is defined on the interval $x\in(22,28)$. Continue $f(x)$ periodically on the entire numerical axis with a period $2l=6$ and make a drawing of a new periodic function $F(x)$; develop a new function $F(x)$ in the Fourier series; set the value of the sum of the Fourier series at the points $x=22\pm 2l\cdot n, n=0,\pm1,\pm2,\dots$. Using the obtained Fourier series, find the sum of the following numerical series: $S=\sum\limits_{n=1}^{\infty}\sin\left(\dfrac{2\pi n}{3}\right)\Big/ n$..</td></tr>
<tr><td colspan="4">4. Using the Fourier cosine transform, solve the integral equation with respect to the function $\varphi(x)$:
$$\int\limits_0^{\infty}\varphi(x)\cos(x\cdot y)dx=\frac{\cos y-\sin y}{e^y},\ y>0.$$</td></tr>
<tr><td colspan="4">5. Given a function $y_0(t)=\begin{cases} t/3, 0\le t\le 3; \\ 2-t/3, 3\le t\le 6. \end{cases}$ on the interval $t\in[0,6]$. Make a graph of the function $y_0(t)$. Then it is extended to the entire numerical axis with the period $T=6$; as a result, we got a periodic signal - the function $y(t)$. Find the Laplace image $Y(p)$ of this new function $y(t)$.</td></tr>
<tr><td colspan="4">6. Find the solution of Volterra's integral equation using the methods of operational calculus and check it:
$$y(t)=\cos 3t-3\int\limits_0^t[\mathrm{ch}[2(t-\tau)]\cdot y(\tau)d\tau.$$</td></tr>
</table>

Lecturer of the academic discipline, prof. ____________ V.P. Legeza