# MATHEMATICAL SUPPORT OF MULTIMEDIA AND INFORMATION RETRIEVAL SYSTEMS. Part 1. Multidimensional integral calculus

## Working program of the academic discipline (Syllabus)

### Details of the academic discipline

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| **Level of higher education** | *First (undergraduate)* |
| **Branch of knowledge** | *12 Information technologies* |
| **Specialty** | *121 Software engineering* |
| **Educational program** | *Software Engineering of Multimedia and Information Retrieval Systems* |
| **Discipline status** | *Normative* |
| **Form of education** | *Daytime* |
| **Year of training, semester** | *Second year of training, third semester* |
| **Scope of the discipline** | *Lectures: 36 hours, practical classes: 36 hours, independent work: 63 hours.* |
| **Semester control** | *Exam, modular control work, calculation and graphic work, calendar control* |
| **Timetable** | *According to the schedule for the autumn semester of the current academic year (rozklad.kpi.ua)* |
| **Language of teaching** | *Ukrainian* |
| **Information about the course leader / teachers** | *Lecturer: doctor of science, professor, Legeza Viktor Petrovych* <br> *legeza@pzks.fpm.kpi.ua* <br> *Practical training: doctor of science, professor, Legeza Viktor Petrovych* |
| **Placement of the course** | *Third semester:* <br> *https://classroom.google.com/c/MzIwMDc3ODQzNjY0?hl=uk&cjc=rb66qqd* |

### Program of study discipline:

#### 1. Description of the educational discipline, its purpose, subject of study and learning outcomes

Study of the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" allows students to develop the competencies necessary for building mathematical models and algorithms in the process of research and solving practical problems of natural science and information technologies.

**The purpose** of studying the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" is the formation of a higher education student's abilities for abstract thinking, independent analysis and synthesis of complex multidimensional systems, as well as the ability to use the acquired fundamental knowledge at the stages of posing a problem in mathematical and symbolic form, followed by its algorithmization and development of modern software software.

**The subject** of the discipline is "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" are methods, techniques and technologies of mathematical analysis and its additional sections, which make up the theoretical justification and mathematical support of the process of solving a wide range of problems belonging to the field of knowledge 12 "Information technologies".

Study of the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" will contribute to the formation of the following **professional competences** (PC):

PC 08 Ability to apply fundamental and interdisciplinary knowledge to successfully solve software engineering problems.

PC 14 Ability to algorithmic and logical thinking.

PC 15 Ability to apply fundamental and interdisciplinary knowledge to build advanced retrieval algorithms.

PC 16 Ability to develop algorithms for implementing statistical data analysis methods.

PC 18 Ability to develop methods for mathematical problems numerical solutions using software.

PC 20 Ability to apply the acquired fundamental mathematical knowledge to develop calculation methods in the multimedia and information retrieval systems creation.

Study of the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" will contribute to the formation of the following **program learning outcomes** (PLO) for students under the EP:

PLO 05 To know and apply relevant mathematical concepts, domain methods, system and object-oriented analysis and mathematical modeling for software development.

PLO 07 To know and to apply in practice the fundamental concepts, paradigms and basic principles of the functioning of language, instrumental and computational tools of software engineering.

PLO 25 To know and to be able to use fundamental mathematical tools in the algorithms construction and modern software development.

PLO 26 To be able to develop and use methods and algorithms for the mathematical problems approximate solution during the multimedia and information retrieval systems design.

PLO 27 To be able to use statistical data analysis methods.

PLO 28 To know the mathematical and algorithmic basics of computer graphics and to be able to apply them to develop multimedia software.

## 2. Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)

Successful study of the discipline (educational component) "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" must be carried out as part of the thoroughly mastered educational material of the disciplines "Mathematical Analysis" and "Linear Algebra and Analytical Geometry" of the first course of bachelor's training in the specialty 121 Software Engineering.

Received during the assimilation of the discipline "Mathematical support of multimedia and information and retrieval systems. Part 1. Multidimensional integral calculus" theoretical knowledge and practical skills are necessary for studying the disciplines "Mathematical support of multimedia and information retrieval systems. Part 2. Fundamentals of harmonic analysis and elements of operational calculus", "Physical foundations of multimedia systems", "Software support of multimedia systems" of the curriculum of bachelor's training in the specialty 121 Software engineering, as well as the disciplines "Operations research and mathematical programming" and "Information retrieval systems and services" of the master's training plan under the EP "Software engineering of multimedia and information retrieval systems".

## 3. Content of the academic discipline.

Discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" involves the study of the following six topics:

Topic 1. Double integral.

Topic 2. Triple integral.
Topic 3. Curvilinear integrals of the 1st and 2nd kind.
Topic 4. Surface integrals of the 1st and 2nd kind.
Topic 5. Elements of field theory. Differential operations of the second order.
Topic 6. Integrals depending on a parameter.
Modular control work (MCW)
Examination

## 4. Educational materials and resources.

### Basic literature


1. S.V.Budak, B.M.Fomin. Multiple Integrals, Field Theory and Series**.** An Advanced Course in Higher Mathematics. Mir Publishers; First printing edition, 1973. – 608 p.
2. V.A.Ilyin and E.G.Pozyak. Fundamentals of mathematical analysis. Part 2. Mir Publishers, 1982. – 438 p.
3. N.Piskunov. Differential and Integral calculus. Vol. 2, CBS Publishers & Distributors, 2021, 572 p.
4. G.M.Fikhtengol'ts. The Fundamentals of Mathematical Analysis (Int. series of Monographs on pure and applied Mathematics). Volume 2. Elsevier, Pergamon Press. 1965, 518 p.
5. B.P.Demidovich. Problems in Mathematical Analysis. Gordon & B., 1969, 496 p.


### Additional literature.


6. Y.B.Zel`dovich, A.D.Myshkis. Elements of Applied Mathematics. Mir Publishers, 1976, 656 p.
7. R.Courant. Differential and Integral Calculus. Vol. 2. Ishi press international, 2010, 682 p.
8. G.N.Berman. A problem book in mathematical analysis. MTG Learning Media (P) Ltd., New Delhi/Gurgaon, 2017, 490 p.
9. V.P. Legeza. Mathematical analysis: a collection of problems. - Kyiv, KPI, Polytechnic Publishing House, 2018. - 240 p.
10. Vladimirov V.M.,Puchkov O.A., Shmygevskyi M.V. A collection of problems in higher mathematics (typical calculations). Part 2. - Kyiv: Polytechnic Publishing House, 2003. -200 p.


## 5. Methods of mastering an educational discipline (educational component)

| № | Educational type occupation | Description of the training session |
|---|---|---|
| | ***Topic 1. Double integral*** | |
| **1** | **Lecture №1**. Double integral: basic concepts and definitions. Area of a flat figure. Squareness of a flat figure. | Double integral: basic concepts and properties. The concept of quadraticity of the region of integration. The border of the region, which has an area (measurement) of "zero". Mira Jordan. A general method of constructing integral sums in a double integral. Conditions of integrability and integrative functions. The most important classes of integral functions.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |
| **2** | **Practical lesson №1.** The technique of reducing the double integral to the repeated one. Right and wrong areas of integration. Arrangement of limits of integration in repeated integrals. | Mastering the technique of reducing a double integral to a repeated one. Correct and incorrect areas of integration. Arrangement of limits of integration in repeated integrals.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |

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| **3** | **Lecture №2.** The technique of reducing the double integral to the repeated one. Properties of the double integral. | Methods of calculating the double integral. The concept of repeated integral. Conditions and technique for reducing a double integral to a repeated integral: a) in rectangular and b) in curvilinear regions.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47<br>To master the concept of the derivative of the additive function of a flat region by area as an analogue of the derivative of the ordinary indefinite integral by the variable of integration. |
| **4** | **Practical lesson №2.** Change of variable in the double integral. Jacobian. Polar coordinates. | Change of variable in the double integral. Jacobian. Polar coordinates.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |
| **5** | **Lecture №3.** Change of variables in the double integral. Calculation of geometric and mechanical characteristics of flat material figures | Change of variable in the double integral. Curvilinear coordinates. The technique of transition from Cartesian rectangular coordinate system to curvilinear coordinates. Area element in curvilinear coordinates. Jacobian and its geometric content. Polar coordinates.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47<br>Construct integral sums: a) to calculate the coordinates of the CM of a non-homogeneous plate; b) to find its moments of inertia when rotating around the coordinate axes. |
| **6** | **Practical lesson №3.** Application of the double integral in geometric and physical problems**.** | Application of the double integral in geometric and physical problems: calculation of the area of a flat region, volumes of bodies, mass of a non-homogeneous flat material figure, coordinates of its CM, moments of inertia relative to coordinate axes.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |
| | ***Topic 2. Triple integral*** | |
| **7** | **Lecture №4.** Triple integral: basic concepts, definitions, properties. The technique of calculating the triple integral. | Triple integral: basic concepts, definitions. Cubic bodies. Upper and lower sums of Darbu. Conditions for the existence of the triple integral. Properties. Integrability of continuous functions. The triple integral as an additive function of the spatial domain $G$. Calculation of the triple integral. Reduction of the triple integral to the double integral if the region of integration is a parallelepiped. Reduction of the triple integral to the repeated integral over a curvilinear region .<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47<br>To study the concept of the derivative of the additive function of the spatial domain by volume as an analogue of the derivative of the ordinary indefinite integral by the variable of integration. |
| **8** | **Practical lesson №4.**<br>Mastering the technique of reducing the triple integral to the repeated one. | Mastering the technique of reducing the triple integral to the repeated one. Right and wrong areas of integration. Arrangement of limits of integration in repeated integrals.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |
| **9** | **Lecture №5**. Change of variables in the triple integral. Application of the triple integral in spatial problems. | Display of spatial areas. Curvilinear coordinates in space. Cylindrical and spherical coordinates. Expression of the volume element in curvilinear coordinates. Change of variables in the triple integral. The geometric content of the Jacobian. Application of the triple integral, examples.Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 – 47.<br>1. Construct integral sums: a) to calculate the coordinates of the CM of a non-homogeneous spatial body; b) to find its moments of inertia when rotating around the coordinate axes.<br>2. Derive the formula for the gravitational interaction of a material point and a material body. |
| **10** | **Practical lesson №5.** Change of variable in the triple integral. Cylindrical and spherical coordinates, Jacobian. | Change of variable in the triple integral. Cylindrical and spherical coordinates, Jacobian and its geometric content.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |
| **11** | **Lecture №6.** Multidimensional integrals. Reintegration formula. Change of variables in multidimensional integrals. | Concept and definition of multidimensional ( $n$ –dimensional) integrals. Reintegration formula. Change of variables in multidimensional integrals. The concept of Jacobian. Theorem justifying the formula for substitution of variables. Spherical |

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| | | coordinates in $n-$dimensional space, Jacobian and volume element for this case.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47<br>Derive the formula for calculating the Jacobian in spherical coordinates in $n-$dimensional space, |
| **12** | **Practical lesson №6. On the first half-pair -** Application of the triple integral in geometric and physical problems.<br>**In the second half,** there is a part of the MCW on Topics No. 1 and No. 2. | The application of the triple integral in geometric and physical problems: calculation of the volume of a spatial region, the mass of a non-homogeneous spatial material figure, the coordinates of its CM, and the moments of inertia relative to the coordinate axes.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 125-139; [10], p.p. 35 - 47 |
| ***Topic 3. Curvilinear integrals of the 1st and 2nd kind*** | | |
| **13** | **Lecture №7**. Curvilinear integral of the first kind (along an arc): definition, properties, calculations, applications. | The concept of a smooth and piecewise-smooth curve and the definition of a curvilinear integral of the first kind as a curvilinear integral over the length of an arc. Physical and geometric meaning of the curvilinear integral of the first kind. Calculation of curvilinear integrals of the first kind for various cases of specifying a curve. Properties of curvilinear integrals of the first kind. Application of curvilinear integrals of the first kind in geometric and physical problems.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 140-149; [10], p.p. 47 - 55<br>Derive the formula for the force interaction of a material point and a material curve. |
| **14** | **Practical lesson №7.** The technique of reducing a curvilinear integral of the 1st kind to an ordinary definite integral. Parameterization of curves. Application to the calculation of the arc length of the curve, mass, CM coordinates and moments of inertia of the material curve | The technique of reducing a curvilinear integral of the 1st kind to an ordinary definite integral. Parameterization of curves. Application to the calculation of the arc length of a curve, its mass, CM coordinates and moments of inertia of a material curve.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 140-149; [10], p.p. 47 - 55 |
| **15** | **Lecture №8**. Curvilinear integral of the second kind (by coordinates): definition, properties, calculation, application. Green's formula. | The concept and definition of a curvilinear integral of the second kind and its physical meaning. Connection between two types of curve integrals. Calculation of the curvilinear integral of the second kind using the definite integral. Dependence of the value of the curvilinear integral of the second kind on the direction of the path of integration. Application of the curvilinear integral of the second kind in geometric and physical problems. Connection of the curvilinear integral of the second kind on the boundary $L$ of the domain $D$ with the double integral on this domain $D$. Green's formula.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 140-149; [10], p.p. 47 - 55<br>1. Prove Green's formula in the case of an irregular domain $D$.<br>2. Using the logic of applying Green's formula, derive your formula for calculating the area of a flat figure. |
| **16** | **Practical lesson №8.** The technique of reducing a curvilinear integral of the second kind to an ordinary definite integral. Using Green's formula to calculate the areas of flat figures. Finding the work of a vector force when moving a material point from one point of space to another. | The technique of reducing a curvilinear integral of the second kind to an ordinary definite integral. Using Green's formula to calculate the areas of flat figures. Finding the work of a vector force when moving a material point from one point of space to another.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 140-149; [10], p.p. 47 - 55 |
| **17** | **Lecture №9**. Conditions for the independence of the curvilinear integral of the second kind from | Four conditions for the independence of the curvilinear integral of the second kind from the path of integration. Recovery of a function by its full differential. Newton-Leibnitz formula. Primary for full differential. Curvilinear integrals in the multi-connected domain. |

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| | the path of integration. Newton-Leibniz formula. | Tasks for **IWS**: [1-3, 5], [9], p.p. 140-149; [10], p.p. 47 - 55<br>1. Calculate the curve integral<br>$$J=\int_{A(1,1)}^{B(2,2)}\left(\operatorname{arctg}\left(\frac{y}{x}\right)+\frac{y^2-xy}{x^2+y^2}\right)dx+\left(\operatorname{arctg}\left(\frac{x}{y}\right)+\frac{x^2-xy}{x^2+y^2}\right)dy,$$<br>showing that its value does not depend on the path of integration. |
| **18** | **Practical lesson №9. On the first half-pair** - Method of recovery of the function by its complete differential. The Newton-Leibniz theorem. Calculation of curvilinear integrals of the 2nd kind along spatial curves.<br>**In the second half**, there is a part of the MCW on topic No. 3. | The method of restoring a function by its complete differential. The Newton-Leibniz theorem. Calculation of curvilinear integrals of the II kind along spatial curves.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 140-149; [10], p.p. 47 - 55 |
| | ***Topic 4. Surface integrals of the 1st and 2nd kind.*** | |
| **19** | **Lecture №10**. Surface integrals of the first kind: basic concepts, definitions, calculation techniques, applications. | Concept and definition of surface integrals of the first kind of a scalar function. Calculation of the area of a curved surface through the double integral. Calculation of the surface integral using the double. A case of surface parameterization. Some applications of surface integrals in physical problems. Surface integrals of the first kind from vector functions. The general concept of the surface integral of the first kind.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62<br>1. Find the moment of inertia of a homogeneous surface of a sphere with a radius $R$ relative to its arbitrary tangent<br>2. To study an example of the gravitational interaction of a material point and a material surface. |
| **20** | **Practical lesson №10.** The technique of reducing the surface integral of the 1st kind to a double over a flat region. The application of surface integrals of the 1st kind to the calculation of the characteristics of material surfaces | The technique of reducing the surface integral of the 1st kind to a double over a flat region. The application of surface integrals of the 1st kind to the calculation of the area of a smooth curved surface, the mass of a material surface, moments of inertia relative to coordinate axes, etc.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62 |
| **21** | **Lecture №11.** Surface integrals of the second kind: basic concepts, definitions, calculation technique through double integrals. | Introductory concepts: oriented (two-sided) and non-oriented surfaces; Möbius sheet, contour orientation. Surface integrals of the second kind: basic concepts, definitions, mechanical content. The flow of the vector field $\vec{F}$ through the surface $\Sigma$. Calculation of the surface integral of the second kind. The general scheme of reducing the calculation of the surface integral of the second kind to the calculation of the double integral. Reduction of the surface integral to the double in the case of a parametric specification of the surface $\Sigma$. Application of surface integrals of the 2nd kind in mechanics.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62<br>1. Find the flow of a vector field<br>$$\vec{F}=(x+2z)\vec{i}+(x+y+2z)\vec{j}+(2x-y)\vec{k}$$<br>through the surface $\Sigma:\{2x+3y+6z=6, x=0, y=0, z=0\}$ in the direction of the external normal to the given surface. |
| **22** | **Practical lesson №11.** Mastering the technique of reducing the surface integral of the II kind to double integrals in three Cartesian | Mastering the technique of reducing the surface integral of the II kind to doubles in three Cartesian planes. Application of surface integrals of the II kind in physics.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62 |

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| | planes. Application of surface integrals of the II kind in physics. | |
| **23** | **Lecture №12.** Formulas of Ostrogradsky and Stokes and their application in practical problems. | Derivation of Ostrogradsky's formula. Calculation of surface integrals using the Ostrogradsky formula. Representation of the volume of a spatial domain in the form of a surface integral. Derivation of the Stokes formula. Application of the Stokes formula for the study of spatial curvilinear integrals.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62<br>Propose your formula, logically similar to Ostrogradsky's formula, for calculating the volume of the region $G$ in the form of a surface integral over the closed surface of the $\Sigma -$boundary of this spatial region $G$. |
| **24** | **Practical lesson №12.** Scalar and vector fields. Calculation of the directional derivative and gradient of a scalar field, the flow of a vector field through a given surface | Scalar and vector fields. Calculation of the directional derivative and gradient of a scalar field, the flow of a vector field through a given surface.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62 |
| ***Topic 5. Elements of field theory*.** | | |
| **25** | **Lecture №13**. Elements of field theory. Scalar and vector fields. Directional derivative and gradient of a scalar field. Vector field flow. | Elements of field theory. Definition and examples of vector fields. Vector lines and vector tubes. Different types of symmetry of vector fields. The vector field of the gradient of the scalar field $U(M)$. Potential fields. Conditions under which a vector field $\vec{F}$ is potential. Vector field flow. Divergence. Formal properties and physical meaning of divergence. Solenoidal vector fields.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62<br>1. To find out the physical meaning of divergence for gravitational and electrostatic fields formed by a certain distribution of masses and charges, respectively.<br>2. Apply the Ostrogradskyi formula when deriving the equation of continuity of the flow of a moving fluid. |
| **26** | **Practical lesson №13.** Calculation of divergence. Ostrogradsky's formula and its application to the calculation of the flow of a vector field through a closed surface. | Calculation of divergence. Ostrogradsky's formula and its application to the calculation of the flow of a vector field through a closed surface.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62 |
| **27** | **Lecture №14.** Vector field circulation Vector field rotor. Stokes' formula written through the vector field rotor. | Concept and definition of circulation and rotor. Properties and physical content of the rotor. Symbolic notation of the formula for calculating the rotor. Stokes formula. The rotor as a rotational component of any vector flow. The connection between the concepts of solenoidality and potential and the rotor.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62<br>1. Explain the physical meaning of the rotor.<br>2. State the condition for the potential of a vector field.<br>3. Prove the formula $\mathrm{div}(\mathrm{rot}\,\vec{F}) = 0$. |
| **28** | **Practical lesson №14.** Calculation of the vector field rotor. Calculation of circulation according to the Stokes formula. Potential and solenoidal vector fields. | Calculation of the vector field rotor. Using the symbolic notation of the matrix formula to calculate the rotor. Calculation of circulation according to the Stokes formula. Potential and solenoidal fields. Verification of the conditions of solenoidality and potential of the vector field.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62 |
| **29** | **Lecture №15**. Elements of vector analysis. Hamilton and Laplace operators. Differential operations of the second order. | Elements of vector analysis. Hamilton and Laplace operators. Differential operations of the second order, the technique of their use for researching the characteristics of vector fields. The spatial equation of thermal conductivity (as an example of the application of differential operations of the second order). Stationary temperature distribution. Harmonic fields. Laplace's equation. |

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| | | Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62<br>1. Derive the spatial equation of thermal conductivity (as an example of the application of differential operations of the second order).<br>2. Show that the function $U(x,y,z)=\frac{k}{r}$, $r=\sqrt{x^2+y^2+z^2}$, $k=const$, defines a harmonic field, i.e. satisfies the Laplace equatin: $\Delta\left(\frac{k}{r}\right)=0$. |
| **30** | **Practical lesson №15. In the first half-pair -** Mastering differential operations of the second order. The use of Hamiltonian and Laplace operators to perform differential operations of the second order.<br>**In the second half,** there is a part of the MCW on topics Nos. 4, 5**.** | Mastery of differential operations of the second order. The use of Hamiltonian and Laplace operators to perform differential operations of the second order.<br>Tasks for **IWS**: [1-3, 5], [9], p.p. 150-159; [10], p.p. 55 - 62**.** |
| | ***Topic 6. Integrals depending on parameters*** | |
| **31** | **Lecture №16.** Eigenintegrals depending on the parameter. Continuity conditions of the integral with respect to the parameter. Conditions for differentiation and integration of the integral by parameter. | Proper integrals depending on the parameter: basic concepts, definitions, properties. Continuity conditions of the integral, as a function of $J(\alpha)$, with respect to the parameter $\alpha$. Conditions under which it is possible to perform differentiation and integration under the sign of the integral $J(\alpha)$ with the parameter $\alpha$. Integrals depending on the parameter, conditions for their integration and differentiation by the parameter $\alpha$ under the sign of the integral.<br>Tasks for **IWS**: [1-2, 4], [5], p.p. 345 - 367**.**<br>1. Calculate the Poisson integral: $J=\int_0^\infty e^{-x^2}dx$.<br>2. Find the integral $J(\alpha)=\int_0^\pi \frac{\ln(1+\alpha\cos x)}{\cos x}dx$, $\lvert\alpha\rvert\le 1$, having previously differentiated by parameter $\alpha$.<br>3. Обчислити інтеграл: $J(u)=\int_0^\infty e^{-kx}\frac{\sin(ux)}{x}dx, k=const>0$. |
| **32** | **Practical lesson №16.** Eigenintegrals depending on the parameter. Using the operations of differentiation and integration of the integral by parameter to calculate complex definite integrals. | Eigenintegrals depending on the parameter. Using the operations of differentiation and integration of the integral by parameter to calculate complex definite integrals.<br>Tasks for **IWS**: [1-2, 4], [5], p.p. 345 - 367**.** |
| **33** | **Lecture №17.** Improper integrals depending on the parameter. Properties of uniformly convergent integrals depending on the parameter. Signs of uniform convergence of improper integrals. | Definition of improper integrals depending on the parameter. The concept of uniform convergence of nonproprietary integrals depending on a parameter. Actions on uniformly convergent improper integrals. Signs of uniform convergence of improper integrals. Weierstrass sign. Cauchy criterion<br>Tasks for **IWS**: [1-2, 4], [5], p.p. 345 - 367**.**<br>1.Calculate the Fresnel integrals using the transition to nonproprietary integrals that depend on the parameter.<br>2. Derive the formula for calculating the Frullani integral<br>$$\int_0^\infty \frac{f(bx)-f(ax)}{x}dx, if\ 0<a<b$$<br>and find the integral: $J=\int_0^\infty \frac{\text{arctg}(bx)-\text{arctg}(ax)}{x}dx, 0<a<b$. |

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| 34 | **Practical lesson №17.** Improper integrals depending on a parameter: definition, properties, uniform convergence and operations on uniformly convergent improper integrals. Use of sufficient signs of uniform convergence of improper integrals (Weierstrass sign, Cauchy criterion) to perform differentiation and integration operations of integrals that depend on the parameter | Improper integrals depending on a parameter: definition, properties, uniform convergence and operations on uniformly convergent improper integrals. Use of sufficient signs of uniform convergence of nonproprietary integrals (Weierstrass sign, Cauchy criterion) to perform differentiation and integration operations of parameter-dependent integrals.<br>Tasks for **IWS**: [1-2, 4], [5], p.p. 345 - 367**.** |
| 35 | **Lecture №18.** Euler's integrals are beta and gamma functions. The technique of calculating definite integrals using Euler's integrals. | Euler integrals – beta and gamma functions: definition, properties, applications. Formula relating beta and gamma functions. Dirichlet's formula. Stirling's formula. Addition formula. The technique of calculating definite integrals using Euler's integrals.<br>Tasks for **IWS**: [1-2, 4], [5], p.p. 345 - 367**.**<br>1. Prove the property of the gamma function: $\Gamma(\alpha+1)=\alpha\Gamma(\alpha)$<br>2. Find: a). $\int_{-1}^{1}\frac{dx}{\sqrt[3]{(1+x)^2(1-x)}}$; б). $\int_{0}^{1}\frac{dx}{\sqrt[n]{1-x^n}}, n>1$; в). $\int_{0}^{\pi/2}\mathrm{tg}^n(x)dx$ |
| 36 | **Practical lesson №18. On the first half pair -** Application of Euler's integrals. The technique of calculating definite integrals using Euler's integrals.<br>**In the second half,** there is a part of the MCW on topic No. 6. | Euler integrals – beta and gamma functions: definition, properties, applications. The technique of calculating definite integrals using Euler's integrals.<br>Tasks for **IWS**: [1-2, 4], [5], p.p. 345 - 367**.** |

## 6. Independent work of the student (IWS)

Mastering the educational material from the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" is based on self-preparation for classroom classes on theoretical and practical topics.

| *№* | *The name of the topic submitted for independent processing* | *Number of hours* | *literature* |
|---|---|---|---|
| *1* | *Preparation for the lecture 1* | *0.5* | [1-3, 5], *Lecture №1,* [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *2* | *Preparation for the practical lesson 1* | *0.5* | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *3* | *Preparation for the lecture 2* | *0.5* | [1-3, 5], *Lecture №2,* [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *4* | *Preparation for the practical lesson 2* | *0.5* | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *5* | *Preparation for the lecture 3* | *0.5* | [1-3, 5], *Lecture №3,* [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *6* | *Preparation for the practical lesson 3* | *0.5* | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *7* | *Preparation for the lecture 4* | *0.5* | [1-3, 5], *Lecture №4,* |

| | | | |
|---|---|---|---|
| | | | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *8* | *Preparation for the practical lesson 4* | *0.5* | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *9* | *Preparation for the lecture 5* | *0.5* | [1-3, 5], *Lecture №5,* [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *10* | *Preparation for the practical lesson 5* | *0.5* | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *11* | *Preparation for the lecture 6* | *0.5* | [1-3, 5], *Lecture №6,* [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *12* | *Preparation for the practical lesson 6* | *0.5* | [9], p.p. 125-139; [10], p.p. 35 – 47 |
| *13* | *Preparation for the lecture 7* | *0.5* | [1-3, 5], *Lecture №7,* [9], p.p. 140-149; [10], p.p. 47-55 |
| *14* | *Preparation for the practical lesson 7* | *0.5* | [9], p.p. 140-149; [10], p.p. 47-55 |
| *15* | *Preparation for the lecture 8* | *0.5* | [1-3, 5], *Lecture №8,* [9], p.p. 140-149; [10], p.p. 47-55 |
| *16* | *Preparation for the practical lesson 8* | *0.5* | [9], p.p. 140-149; [10], p.p. 47-55 |
| *17* | *Preparation for the lecture 9* | *0.5* | [1-3, 5], *Lecture №9,* [9], p.p. 140-149; [10], p.p. 47-55 |
| *18* | *Preparation for the practical lesson 9* | *0.5* | [9], p.p. 140-149; [10], p.p. 47-55 |
| *19* | *Preparation for the lecture 10* | *0.5* | [1-3, 5], *Lecture №10,* [9], p.p. 150-159; [10], p.p. 55-62 |
| *20* | *Preparation for the practical lesson 10* | *0.5* | [9], p.p. 150-159; [10], p.p. 55-62 |
| *21* | *Preparation for the lecture 11* | *0.5* | [1-3, 5], *Lecture №11,* [9], p.p. 150-159; [10], p.p. 55-62 |
| *22* | *Preparation for the practical lesson 11* | *0.5* | [9], p.p. 150-159; [10], p.p. 55-62 |
| *23* | *Preparation for the lecture 12* | *0.5* | [1-3, 5], *Lecture №12,* [9], p.p. 150-159; [10], p.p. 55-62 |
| *24* | *Preparation for the practical lesson 12* | *0.5* | [9], p.p. 150-159; [10], p.p. 55-62 |
| *25* | *Preparation for the lecture 13* | *0.5* | [1-3, 5], *Lecture №13,* [9], p.p. 150-159; [10], p.p. 55-62 |
| *26* | *Preparation for the practical lesson 13* | *0.5* | [9], p.p. 150-159; [10], p.p. 55-62 |
| *27* | *Preparation for the lecture 14* | *0.5* | [1-3, 5], *Lecture №14,* [9], p.p. 150-159; [10], p.p. 55-62 |
| *28* | *Preparation for the practical lesson 14* | *0.5* | [9], p.p. 150-159; [10], p.p. 55-62 |
| *29* | *Preparation for the lecture 15* | *0.5* | [1-3, 5], *Lecture №15,* [9], p.p. 150-159; [10], p.p. 55-62 |

| | | | |
|---|---|---|---|
| *30* | *Preparation for the practical lesson 15* | *0.5* | [9], p.p. 150-159; [10], p.p. 55-62 |
| *31* | *Preparation for the lecture 16* | *0.5* | [1-2, 4], *Lecture №16,* [5], p.p. 345 - 367**.** |
| *32* | *Preparation for the practical lesson 16* | *0.5* | [5], p.p. 345 - 367**.** . |
| *33* | *Preparation for the lecture 17* | *0.5* | [1-2, 4], *Lecture №17,* [5], p.p. 345 - 367 |
| *34* | *Preparation for the practical lesson 17* | *0.5* | [5], p.p. 345 - 367 |
| *35* | *Preparation for the lecture 18* | *0.5* | [1-2, 4], *Lecture №18,* [5], p.p. 345 - 367 |
| *36* | *Preparation for the practical lesson 18* | *0.5* | [5], p.p. 345 - 367 |
| *37* | *Preparation for MCW* | *5* | [1-4], [5], p.p. 345 – 367; [9], p.p. 125-159 [10], p.p. 35-62 |
| *38* | *Preparation for IWS* | *10* | [1-4], [5], p.p. 345 – 367; [9], p.p. 125-159 [10], p.p. 35-62 |
| *39* | *Preparation for the exam* | *30* | [1-4], [5], p.p. 345 – 367; [9], p.p. 125-159 [10], p.p. 35-62 |

## Policy and control

### 7. Policy of academic discipline (educational component)

1. **General policy of teaching** the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" is aimed at independent performance of educational tasks, tasks of current and final control of learning results (for persons with special educational needs, this requirement is applied taking into account their individual needs and capabilities); mandatory correct reference to sources of information in the case of using other people's ideas, developments, statements, technologies; providing reliable information about the results of one's own educational (scientific, creative) activities, used research methods, technologies and sources of information,
2. **Visitation Policy**. In the normal course of study, attendance at both lectures and practical classes is a mandatory component of assessment. In long-term force majeure circumstances (military actions, pandemics, international internships), training can be conducted remotely. In this case, the absence of a classroom lesson does not involve the calculation of penalty points, since the student's final rating score is formed exclusively during the final examination. At the same time, independent performance of modular control tasks and defense of individual thematic tasks, as well as speeches (reports) at colloquiums and active work in practical classes will be evaluated during classroom classes.
3. **Policy on working out and redoing assessment control measures**. According to the regulation "Regulations on current, calendar and semester control of study results at Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf) every student has the right to make up for classes missed for a good reason and assessment control measures (hospital, mobility, etc.) at the expense of independent work.
4. **The procedure for contesting the results of assessment control measures**. According to the "Regulations on the resolution of conflict situations in the Igor Sikorsky Kyiv Polytechnic Institute" (https://osvita.kpi.ua/node/169) students have the right to challenge the results of the control measures with arguments, explaining which criterion they disagree

with according to the assessment. A student may raise any issue relating to the assessment procedure and expect it to be dealt with in accordance with pre-defined procedures.

5. **Academic integrity**. The policy and principles of academic integrity are governed by the norms set forth in Chapter 3 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).
6. **Norms of ethical behavior**. The norms of ethical behavior of students and scientific and pedagogical workers are regulated by the provisions set forth in Chapter 2 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).
7. **Inclusive education**. Acquisition of knowledge and skills in the course of studying the discipline "Mathematical support of multimedia and information retrieval systems. Part 1: Multidimensional Integral Calculus' is accessible to most people with special educational needs, except for those with severe visual impairments who cannot complete tasks using personal computers, laptops and/or other technical aids.
8. **Calendar control** is carried out in order to improve the quality of students' education and monitor the student's fulfillment of the syllabus requirements. Read more: Chapter 3 "Regulations on current, calendar and semester control of study results at the Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf).
9. **Studying in a foreign language**. In the process of mastering the lecture material and performing practical tasks in the discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus" students are recommended to refer to the English-language sources listed in the lists of basic and additional literature.
10. **Assignment of incentive and penalty points**. In accordance with the "Regulations on the system of evaluation of learning results at the Igor Sikorsky Kyiv Polytechnic Institute" the sum of all incentive points cannot exceed 10% of the rating scale (https://osvita.kpi.ua/node/37). The rules for assigning incentive and penalty points are as follows.

    **Incentive points** are awarded for: a) writing theses, articles, design of a new mathematical problem/technology as a scientific work for participation in a competition of student scientific works (on the subject of the academic discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus") – up to 2 points; b) participation in international or all-Ukrainian events and competitions (on the subject of the academic discipline "Mathematical support of multimedia and information retrieval systems. Part 1. Multidimensional integral calculus") - up to 3 points.

    **Penalty points** are awarded for violations of the principles of academic integrity (non-independent performance of MCW and IWS, writing off during the exam): - 5 points for each violation (attempted plagiarism).

    Self-examination, preparation for practical classes, performance of individual tasks and control measures are carried out during independent work of students with the possibility of consulting with the teacher at the specified consultation time or by means of electronic correspondence (e-mail, messengers).

## 8. Types of control and rating system for evaluating learning outcomes (ELO)

Rating of the discipline "Mathematical support of multimedia and information and retrieval systems. Part 1. Multidimensional integral calculus" consists of:
1) points for individual calculation and graphic work (IWS),
2) points for the integrated modular control work (MCW),
3) points for answering the exam,
4) incentive points,

5) penalty points.

## RATING SYSTEM OF EVALUATION (ELO)

### 8.1 Points for the implementation and protection of an individual IWS.

During each semester, students perform one individual IWS, which is divided into all thematic sections.

**The maximum number of points for a semester individual IWS: 20 points (total).**

Points are awarded for:
- quality of execution (for individual IWS): 0-8 points;
- answer during the defense (for individual IWS): 0-8 points;
- timely submission of work for defense: 0-4 points.

Performance evaluation criteria:
8 points – the work is done qualitatively, in full;
6 points - the work is done qualitatively, in full, but has shortcomings;
3 points - the work is completed in full, but contains minor errors;
0 points – the work is incomplete or contains significant errors.
Criteria for evaluating the quality of the answer:
8 points – the answer is complete, well-argued;
6 points – the answer is generally correct, but has flaws or minor errors;
3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Criteria for evaluating the timeliness of work submission for defense:
4 points – the work is presented for defense no later than the specified deadline;
2 points – the work is submitted for defense 1 week later than the specified deadline;
0 points – the work is submitted for defense more than 1 week later than the specified deadline.
The maximum number of points for the implementation and defense of the semester individual IWS: 8+8+4=20 points (in total for three thematic sections).

### 8.2 Points for the completion of the semester integrated modular control work (MCW).

During the semester, students complete one semester-long integrated modular test, divided by thematic sections into four equal parts in terms of points and time; all tasks are written, including one theoretical and three practical.

**The maximum total number of points for the integrated semester MCW is 30 points**.

**Criteria for evaluating written tasks of the integrated MCW**:
30 points - the solution of the MCW tasks is absolutely (100%) correct;
27 points - the solution of the vast majority of tasks is correct, but in 10% of the tasks there are insignificant errors;
24 points – most tasks are solved correctly, but 20% of the tasks contain errors;
10-15 points - half of the tasks are solved correctly, but 50% of the tasks have significant errors;
5-6 points - the solution of 20% of the tasks is correct, but there are significant errors in 80% of the tasks;
1-3 points – the solution of 10% of tasks is correct, but 90% of tasks have significant errors;
0 points - there is no answer or the answer is 100% incorrect.

The semester component of the rating scale: $\mathbf{R_S = R_{IWS} + R_{MCW} = 20+30}$ **points = 50 points**.

### 8.3. Penalty points.

Penalty points are calculated for:
- academic dishonesty (plagiarism, non-independent performance of MCW, IWS, etc.) - 5 points for one attempt.

### 8.4. Points for answers on the exam.

The examination ticket consists of 6 questions - 1 theoretical and 5 practical. The answer to a theoretical question is worth 10 points, and the answer to each practical question is worth 8 points.
Evaluation criteria for the theoretical question of the examination paper:

10 points – the answer is correct, complete, well-argued;
8-9 points – the answer is correct, detailed, but not very well argued;
6-7 points - in general, the answer is correct, but has flaws;
4-5 points – there are minor errors in the answer;
1-3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Evaluation criteria for the practical question of the examination work:
8 points – the answer is correct, the calculations are completed in full;
6-7 points - the answer is correct, but not very well supported by calculations;
5 points - in general, the answer is correct, but has flaws;
3-4 points – there are minor errors in the answer;
1-2 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.

**The maximum number of points for an answer on the exam:**
**$R_E$ = 10 points × 1 theoretical question + 8 points × 5 practical tasks = 50 points.**

**8.5. Calculation of the rating scale (R).**
The semester component of the rating scale $R_S$= 50 points, it is defined as the sum of positive points received for the completion of the integral modular control work, for the completion and defense of the individual IWS and negative penalty points.
The examination component of the rating scale is equal to: $R_E$ = 50 points.
**The rating scale for the discipline is equal to: R = $R_S$ + $R_E$ = 100 points**.

**8.6. Calendar control: is carried out twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements**:
At the first certification (8th week), the student receives "credited" if his current rating is at least 10 points (50% of the maximum number of points a student can receive before the first certification).
At the second certification (14th week), the student receives "passed" if his current rating is at least 20 points (50% of the maximum number of points a student can receive before the second certification).
**8.7. Conditions for admission to the exam and determining the grade**.
A necessary condition for a student's admission to the exam is the completion and defense of all individual works and the student's semester rating of at least 60% of the $R_S$, i.e. at least 30 points. Otherwise, the student must do additional work and improve his rating.
The total rating of the R student is defined as the sum of the semester rating of the student $R_S$ and the $R_E$ points received on the exam. The grade is assigned according to the value of R according to the table. 1.

**Table 1**

| **Total rating $R_D$** | **Grade** |
|---|---|
| 95-100 | excellent |
| 85-94 | very good |
| 75-84 | good |
| 65-74 | satisfactory |
| 60-64 | enough |
| **$R_D$** ≤ 59 | unsatisfactory |
| **$r_C$** < 30 or not performed (not protected) all types of work. | not allowed |

**8.8 Additional information on the discipline (educational component)**
A copy of the typical exam tickets issued for each semester control is given in Appendix 1.

Working program of the academic discipline (syllabus):
**Compiled by DcS., prof. Legeza V.P.**
**Adopted by** Computer Systems Software Department (protocol № 12 from 26.04.23)

**Approved by** the Faculty Board of Methodology (protocol № 10 from 26.05.23)

**Appendix 1**

**TYPICAL EXAMINATION TICKET**
**FROM THE DISCIPLINE "MATHEMATICAL SUPPORT OF MULTIMEDIA AND INFORMATION RETRIEVAL SYSTEMS-1"**
**FOR SEMESTER CONTROL (THIRD SEMESTER)**

<table>
<tr><td colspan="4">НАЦІОНАЛЬНИЙ ТЕХНІЧНИЙ УНІВЕРСИТЕТ УКРАЇНИ «КПІ» ім. І.Сікорського</td></tr>
<tr><td>Educational and qualification level Bachelor Specialty 121 "Software Engineering"</td><td>Department of software of computer systems<br><br>2022 – 2023 education year</td><td>EXAMINATION TICKET No. 1 from the academic discipline<br><br>MATHEMATICAL SUPPORT OF MULTIMEDIA AND INFORMATION RETRIEVAL SYSTEMS-I 3rd semester</td><td>I approve Chief Department of software of computer systems<br>________________<br>(signature)<br>DcS., Assoc.prof.<br>E.S. Sulema<br><br>Prot. No. 13 dated 06/22/2022</td></tr>
<tr><td colspan="4">Examination theoretical questions</td></tr>
<tr><td colspan="4">1. Replacement of variables in the triple integral. Display of spatial areas. Jacobian. Curvilinear coordinates in space, their double meaning.</td></tr>
<tr><td colspan="4">Practical tasks of various types</td></tr>
<tr><td colspan="4">2. Find the mass of a body $G$ bounded by surfaces $\{x^2+y^2+z^2=3a^2;\ x^2+y^2=2az;\ a>0\}$ if the density at each of its points is equal to the sum of the squares of the coordinates of the point.</td></tr>
<tr><td colspan="4">3. Find the circulation of the vector field $\vec{F}=-4x^2y\vec{i}-3xy^2\vec{j}+z^2\vec{k}$ along the contour $C$ formed by the intersection of the plane and the cylinder $C:\{x+2y-2z=6; x^2+z^2=4\}$. Select the side of the plane for which the normal forms an obtuse angle with the appliqué axis. Make a contour drawing.</td></tr>
<tr><td colspan="4">4. Find the flow of the vector field $\vec{F}=(5x^2+2yz^2)\vec{i}+(3y^2-3xz^2)\vec{j}+(2z^2+7x^2y)\vec{k}$ through the complete outer side of the surface of the pyramid formed by the planes $\Pi:\{-x+2y+2z+4=0, x=0; y=0, z=0\}$. Make a picture.</td></tr>
<tr><td colspan="4">5. Calculate the integral depending on the parameters: $\int_0^1 \sin\left[\ln\left(\frac{1}{x}\right)\right]\cdot\frac{x^\beta-x^\alpha}{\ln x}dx,\ \alpha>0, \beta>0$. Justify the answer. Using this result, calculate the integral: $\int_0^1 \sin\left[\ln\left(\frac{1}{x}\right)\right]\cdot\frac{x^2-1}{\ln x}dx$.</td></tr>
<tr><td colspan="4">6. Using Euler's integrals, find the integral: $J=\int_0^1\frac{dx}{\sqrt[6]{1-x^6}}$.</td></tr>
</table>

**Lecturer of the academic discipline, prof. ____________ V.P. Legeza**