MATHEMATICAL ANALYSIS. Part 2. Differential and integral calculus of functions of many variables

Working program of the academic discipline (Syllabus)

Details of the academic discipline

Level of higher education	***First (undergraduate)***
Branch of knowledge	***F Information technologies***
Specialty	***F2 Software engineering***
Educational program	***Software engineering of multimedia and information retrieval systems***
Discipline status	***Normative***
Form of education	***Daytime***
Year of training, semester	***First year of training, second semester***
Scope of the discipline	***Lectures: 30 hours, practical classes: 30 hours, independent work: 90 hours.***
Semester control	***Exam, modular control work, calculation and graphic work, calendar control***
Timetable	***According to the schedule for the spring semester of the current academic year (rozklad.kpi.ua)***
Language of teaching	***English***
Information about the course leader / teachers	***Lecturer: doctor of science, professor, Legeza Viktor Petrovych*** ***legeza@pzks.fpm.kpi.ua*** ***Practical training: Ph.D., associate professor, Oleksandr Mykhailovych Neshchadym, om.neshchadym@gmail.com***
Placement of the course	***second semester:*** ***https://classroom.google.com/w/MjY4ODc3OTMyOTM5/t/all***

Program of study discipline:

1. Description of the educational discipline, its purpose, subject of study and learning outcomes

Study of the discipline “Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables” allows students to develop the competencies necessary for building mathematical models and algorithms in the process of research and solving practical problems of natural science and information technologies.

The purpose of studying the discipline “Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables” is the formation of a higher education student's abilities for abstract thinking, independent analysis and synthesis of complex multidimensional systems, as well as the ability to use the acquired fundamental knowledge at the stages of posing a problem in mathematical and symbolic form, followed by its algorithmization and development of modern software.

The subject of the discipline is “Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables” are methods, techniques and technologies of mathematical analysis and its additional sections, which make up the theoretical justification and mathematical support of the process of solving a wide range of problems belonging to the field of knowledge F "Information technologies".

Study of the discipline “Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables” will contribute to the formation of the following **general competencies** (GC):

GC01 Aptitude for abstract thinking, analysis and synthesis.

GC02 Ability to apply knowledge in practical situations.

GC03 Ability to communicate in the national language both orally and in writing.

GC05 Ability to learn and master up-to-date knowledge.

GC06 Ability to search, process and analyze information from various sources.

GC08 Ability to act on the basis of ethical considerations.

GC13 Ability to make decisions and act in accordance with the principle of inadmissibility of corruption and any other manifestations of dishonesty.

And professional competencies (PC):

PC08 Ability to apply fundamental and interdisciplinary knowledge to successfully solve software engineering problems.

PC14 Aptitude for algorithmic and logical thinking.

PC15 Ability to apply fundamental and interdisciplinary knowledge to build advanced algorithms for multimedia data processing and retrieval.

PC16 Ability to apply fundamental and interdisciplinary knowledge to build advanced algorithms for reproducing typical physical processes in multimedia and immersive systems.

PC22 Ability to apply and develop object tracking algorithms based on artificial intelligence technologies in multimedia and immersive systems.

Study of the discipline “Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables” will contribute to the formation of the following **program learning outcomes** (PLO) for students under the EP:

PLO01 To analyze, purposefully search for and select the information and reference resources and knowledge necessary for solving professional tasks, taking into account modern achievements of science and technology.

PLO05 To know and apply relevant mathematical concepts, methods of domain, system and object-oriented analysis and mathematical modeling for software development.

PLO07 Know and apply in practice the fundamental concepts, paradigms and basic principles of the functioning of linguistic, instrumental and computing tools of software engineering.

PLO10 To conduct a pre-project survey of the subject area, system analysis of the design object.

PLO11 To choose the initial data for design, guided by formal methods of describing requirements and modelling.

PLO18 To know and be able to apply information technologies for data processing, storage and transmission.

PLO26 To know and be able to use fundamental mathematical tools when building algorithms for processing multimedia data, retrieval, and reproducing typical physical processes in information retrieval, multimedia, and immersive systems.

PLO35 To know and be able to use software tools and multimedia data processing technologies to develop software for information retrieval, multimedia, and immersive systems.

2. Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)

Successful study of the discipline "Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables " should be provided as part of the thoroughly mastered educational material of the discipline "Mathematical analysis. Part 1. Differential and integral calculus of functions of one variable", as well as the discipline "Linear algebra and analytical geometry".

Received during mastering the discipline "Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables" theoretical knowledge and practical skills are necessary for studying the disciplines "Probability theory", "Mathematical support of multimedia and information retrieval systems", "Software of multimedia systems" of the curriculum of bachelor's training in the specialty F2 Software engineering, as well as the disciplines "Operations Research and Mathematical Programming" and "Mathematical modeling of systems and processes " of the curriculum of the master of science training under the EP "Software engineering of multimedia and information retrieval systems".

3. Content of the academic discipline.

Discipline "Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables" involves the study of the following topics:

Topic 1. Boundary, continuity, differentiability of FMV.
Topic 2. Application of partial derivatives.
Topic 3. Double integrals.
Topic 4. Triple integrals.
Topic 5. Number series. Signs of convergence.
Topic 6. Power series.
Modular control work (MCW).
Exam

4. Educational materials and resources

Basic literature

1. G. M. Fikhtengol'ts, The Fundamentals of Mathematical Analysis. (International Series of Monographs in Pure and Applied Mathematics, Vol. 72/73) Vol. 1: 491 p. Oxford/London/Edinburg/New York/Paris/Frankfurt 1965.
2. G. M. Fikhtengol'ts, The Fundamentals of Mathematical Analysis. (International Series of Monographs in Pure and Applied Mathematics, Vol. 72/73) Vol. 2: 518 p. Oxford/London/Edinburg/New York/Paris/Frankfurt 1965.
3. S. M. Nikolsky. Course in Mathematical Analysis. Vol. 1: Mir Publishers, 1977. 460 p.
4. V.A.Ilyin and E.G.Pozyak. Fundamentals of mathematical analysis. Part 1. Mir Publishers, 1982, 637 p. https://www.biblio.com/book/fundamentals-mathematical-analysis-ilyin-v-e/d/734370748.
5. V.A. Ilyin and E.G. Pozyak. Fundamentals of mathematical analysis. Part 2. Mir Publishers, 1982, (438 p.). https://www.biblio.com/book/fundamentals-mathematical-analysis-ilyin-v-e/d/734370748.
6. B.P. Demidovich. Problems in Mathematical Analysis. Gordon & B., 1969, 496 p.
7. V.P. Legeza. Mathematical analysis: a collection of problems. - Kyiv, KPI, Polytechnic Publishing House, 2018. - 240 p.
8. V.P. Legeza, O.M. Neshchadym. Workshop on mathematical analysis. Study guide. Part 2. – Kyiv, Igor Sikorsky Kyiv Polytechnic Institute. - 2024. - 344 p.
9. Google classroom: Mathematical analysis workshop (second semester). https://classroom.google.com/w/MjY4ODc3OTMyOTM5/t/all

Additional literature.

10. S.V.Budak, B.M.Fomin. Multiple Integrals, Field Theory and Series**.** An Advanced Course in Higher Mathematics. Mir Publishers; First printing edition, 1973. – 608 p. https://books.google.com.ua/books/about/Multiple_Integrals_Field_Theory_and_Seri.html?id=XKQNAQAAIAAJ&redir_esc=y
11. Y.B.Zel`dovich, A.D.Myshkis. Elements of Applied Mathematics. Mir Publishers, 1976, 656 p.
12. R.Courant. Differential and Integral Calculus. Vol. 2. Ishi press international, 2010, 682 p.
13. G.N.Berman. A problem book in mathematical analysis. MTG Learning Media (P) Ltd., New Delhi/Gurgaon, 2017, 490 p.
14. N.Piskunov. Differential and Integral calculus. Vol. 1,2, Mir Publishers, 1969. 895 p. https://mmsallaboutmetallurgy.com/wp-content/uploads/2019/01/differential-and-integral-calculus-by-piskunov.pdf.

5. Methods of mastering an educational discipline (educational component)

№	Educational type occupation	Description of the training session
	Section I. Differential calculus FMV. ***Topic 1. Limit, continuity, differentiability of FMV***	
1	**Lecture №1**. Definition of FMV and limits of the sequence of points $\{M_k\}$ of Euclidean space E_n. Lemma about coordinate convergence of a sequence of points in Euclidean space. Cauchy criterion. Bolzano-Cauchy theorem for FMV.	Definition of FMV. The limit of a sequence of points in Euclidean space. Lemma about coordinate convergence of a sequence of points in Euclidean space. The concept of a fundamental sequence of points in Euclidean space. Cauchy criterion. Bolzano-Cauchy theorem for FMV. The concept and two definitions of the FMV limit according to Heine and Cauchy. Theorem on arithmetic operations on the limits of FMV. Tasks for **IWS**: p.6, №1
2	**Practical lesson №1.** The limit of a sequence of points in Euclidean space. Operations on infinitely small FMV. The main and repeated limits of FMV. Investigation of FMV for continuity. Investigation of FMV for uniform continuity according to Cantor.	Graphs of a function of two variables. Construction of isolines and isosurfaces. The limit of a sequence of points in Euclidean space. The concept of a fundamental sequence of points in Euclidean space. Technique for calculating the limits of a function FMV. Operations on infinitely small functions. Fundamental and repeated limits. Investigation of a function FMV for continuity. Investigation of a function FMV for uniform continuity according to Cantor. Tasks for **IWS**: p.6, №2
3	**Lecture №2.** Infinitesimal FMV. Cauchy criterion. Repeated limits. Continuity of FMV. Properties of continuous functions.	Definition of infinitesimal FMV. Necessary and sufficient conditions for the existence of the FMV limit. The concept of the main and repeated limits of FMV. Sufficient conditions for the existence and equality of the repeated limits of the FMV. Concept and three definitions of FMV continuity. Properties of continuous functions. Two Theorems of Weierstrass. Uniform continuity. Cantor's theorem. Tasks for **IWS**: p.6, №3
4	**Practical lesson №2.** The technique of finding partial derivatives of FMV. Research of FMV on differentiability. Using the first differential for approximate calculations.	The technique of finding partial derivatives of FMV. Research of FMV on differentiability. Using the first differential for approximate calculations. Tasks for **IWS**: p.6, №4
5	**Lecture №3.** Partial derivatives of FMV. The concept of differentiability of FMV. The first differential and its application to approximate calculations.	Definition of partial derivatives. Partial derivatives of higher orders. Schwartz's theorem. Definition of differentiability of FMV, necessary and sufficient conditions for differentiability of function of two variables. The fundamental difference between the differentiation of FMV and the differentiation of FOV. The first differential of a function of two variables, its connection with the existence of partial derivatives and application to approximate calculations. Tasks for **IWS**: p.6, №5
6	**Practical lesson №3.** The technique of finding differentials of higher orders. The derivative of the composite FMV. Invariance of the form of the complete differential. The technique of differentiating implicit functions. Taylor's formula for a function of two variables.	The technique of finding differentials of higher orders. The derivative of the composite FMV. Invariance of the form of the complete differential. The technique of differentiation of implicit functions. Taylor's formula for a function of two variables. Tasks for **IWS**: p.6, №6
7	**Lecture №4.** Differentials of higher orders. The derivative of the composite FMV. Invariance of the form of the complete differential. Differentiation of implicit functions. Taylor's formula for a function of two variables.	Differentials of higher orders. The formula for the differential of $n-$th order. The derivative of the composite FMV. Full derivative. Invariance of the form of the complete differential. Theorems on the existence of implicit functions of one and two variables. Differentiation of implicit functions. Taylor and Maclauren's formula for a function of two variables. The residual term of Taylor's formula in Lagrange form. Tasks for **IWS**: p.6, №7
8	**Practical lesson №4.** Application of partial derivatives: tangent plane and normal to the surface. Derivative in direction. Gradient and its properties.	Tangent plane and normal to the surface. Derivative in direction. The physical content of the gradient and its properties. Tasks for **IWS**: p.6, №8

***Topic 2. Application of partial derivatives*.**		
9	**Lecture №5.** Tangent plane and normal to the surface. Derivative in direction. Gradient and its properties.	Definition of the tangent plane and the normal to the surface. The geometric meaning of the differential of a function of two variables. Definition of a scalar field. Types of scalar fields. Derivative in direction, its physical meaning. Concept of gradient, its properties. The connection of the gradient with the derivative in the direction. Tasks for **IWS**: p.6, №9
10	**Practical lesson №5.** Local extremum of FMV. The largest and smallest value of a continuous function FMV in a closed bounded region.	Study of the local extremum of FMV. A technique for finding the largest and smallest value of a continuous function FMV in a closed bounded domain. Tasks for **IWS**: p.6, №10
11	**Lecture №6.** Local extremum of FMV. The largest and smallest value of a continuous function in a closed bounded region. Conditional extremum. Lagrange function and multipliers. The method of least squares.	Local extremum of a function of two variables. Stationary and critical points of the function. Necessary and sufficient conditions for the existence of a local extremum. A rule for investigating a function of two variables for a local extremum. Concept of quadratic forms. Sylvester criterion. The largest and smallest value of a continuous function in a closed and bounded region. Conditional extremum. Lagrange function and multipliers. The method of least squares. Regression. Tasks for **IWS**: p.6, №11
12	**Practical lesson №6.** ***On the first half-pair:*** Conditional extremum. Lagrange function and multipliers. The method of least squares. Regression. ***In the second half:*** the first part of the MCW on topics #1-2.	***On the first half-pair*** Conditional extremum. Lagrange function and multipliers. The method of least squares. Regression. ***In the second half:*** the first part of the MCW on topics #1-2. Tasks for **IWS**: p.6, №12
Section II. Multiple Integrals. ***Topic 3. Double Integrals.***		
13	**Lecture №7**. Double integral: basic concepts and definitions. Area of a plane figure. Quadrature of a plane figure.	Double integral: basic concepts and properties. The concept of quadrature of the integration domain. The boundary of a domain with an area of measure "zero". General methodology for constructing integral sums in a double integral. Darboux sums. Integrability conditions and integrability functions. The most important classes of integrable functions. Tasks for **IWS**: p.6, №13 1. The concept of the measure of sets. Basic properties of area.
14	**Practical lesson №7.** Technique of reducing a double integral to a repeated integral. Correct and incorrect domains of integration. Setting the limits of integration in repeated integrals.	Mastering the technique of reducing a double integral to a repeated integral. Correct and incorrect domains of integration. Setting the limits of integration in repeated integrals. Tasks for **IWS**: p.6, №14
15	**Lecture №8**. Technique of reducing a double integral to a repeated integral. Properties of a double integral.	Methods of calculating a double integral. The concept of a repeated integral. Conditions and techniques for reducing a double integral to a repeated integral: a) in a rectangular and b) in a curvilinear domain. Tasks for **IWS**: p.6, No. 15; 1. Master the concept of the derivative of an additive function of a flat domain with respect to area as an analogue of the derivative of an ordinary indefinite integral with respect to the integration variable.
16	**Practical lesson №8.** Change of variable in a double integral. Jacobian. Technique of integration in polar coordinates.	Change of variable in a double integral. Jacobian. Polar coordinates. Tasks for **IWS**: p.6, No. 16;
17	**Lecture №9**. Substitution of variables in a double integral. Calculation of geometric and mechanical characteristics of flat material figures	Substitution of a variable in a double integral. Curvilinear coordinates. Technique for transition from a Cartesian rectangular coordinate system to curvilinear coordinates. Area element in curvilinear coordinates. Jacobian and its geometric meaning. Polar coordinates. Calculation of geometric and mechanical characteristics of flat material figures.

		1. Construct integral sums: a) to calculate the coordinates of the CM of an inhomogeneous plate; b) to find its moments of inertia when rotating around the coordinate axes. Tasks for **IWS**: p.6, No.17;
18	**Practical lesson №9.** Application of the double integral in geometric and physical problems.	Application of the double integral in geometric and physical problems: calculation of the area of a flat region, volumes of bodies, mass of a non-uniform flat material figure, coordinates of its CM, moments of inertia relative to coordinate axes. Tasks for **IWS**: p.6, No.18;
	Topic 4. Triple integrals.	
19	**Lecture №10**. Triple integral: basic concepts, definitions, properties. Technique for calculating the triple integral.	Triple integral: basic concepts, definitions. Cubic bodies. Upper and lower Darboux sums. Conditions for the existence of a triple integral. Properties. Integrability of continuous functions. Triple integral as an additive function of a spatial domain. Calculation of a triple integral. Reduction of a triple integral to a repeated integral in the case when the integration domain is a parallelepiped. Reduction of a triple integral to a repeated integral over a curvilinear domain. Tasks for **IWS**: p.6, No.19;
20	**Practical lesson №10.** Mastering the technique of reducing a triple integral to an iterated integral.	Reduction of a triple integral to a repeated integral. Correct and incorrect domains of integration. Setting the limits of integration in repeated integrals. Tasks for **IWS**: p.6, No.20;
21	**Lecture №11.** Change of variables in a triple integral. Application of the triple integral in spatial problems. Concept of multidimensional integrals. Reintegration formula. Change of variables in multidimensional integrals.	Mapping spatial regions. Curvilinear coordinates in space. Cylindrical and spherical coordinates. Expression of a volume element in curvilinear coordinates. Change of variables in a triple integral. Geometric meaning of the Jacobian. Application of the triple integral, examples. Concept and definition of multidimensional ($n-$dimensional) integrals. Reintegration formula. Change of variables in multidimensional integrals. Concept of the Jacobian. Theorem justifying the formula for the change of variables. Spherical coordinates in a $n-$dimensional space, the Jacobian and the volume element for this case. 1. Construct integral sums: a) to calculate the coordinates of the CM of an inhomogeneous spatial body; b) to find its moments of inertia when rotating around the coordinate axes. 2. Derive the formula for the gravitational interaction of a material point and a material body. 3. Derive the formula for calculating the Jacobian in spherical coordinates in a $n-$dimensional space. Tasks for **IWS**: p.6, No. 21;
22	**Practical lesson №11. On the first half-pair** - Application of the triple integral in geometric and physical problems. **On the second half-pair** - part of the MCR on Topics No. 3 and No. 4.	Application of the triple integral in geometric and physical problems: calculation of the volume of a spatial region, the mass of a non-uniform spatial material figure, the coordinates of its CM, and the moments of inertia relative to the coordinate axes. Calculation of the volume of a sphere in a $n-$dimensional space. Tasks for **IWS**: p.6, No. 22;
	Section III. Series ***Topic 5. Number series. Signs of convergence.***	
23	**Lecture № 12.** Number series and their relationship with number sequences. Cauchy criterion. Properties of numerical series. Positive numerical series and sufficient signs of their convergence.	Number series: basic concepts and definitions. Partial sum and the concept of convergence of numerical series. The study of numerical series as a new form of studying the properties of numerical sequences. The Cauchy criterion for the convergence of a numerical series. The simplest properties of number series. Remainder of a number series. A necessary condition for the convergence of the numerical series. A sufficient condition for the divergence of a number series. Sufficient signs of convergence of positive number series: sign of comparison by inequality and limit sign; Dalambert's sign, Cauchy's radical sign, Cauchy-McLauren's integral sign; signs of Raabe and Gauss. Tasks for **IWS**: p.6, No. 23;
24	**Practical lesson №12.** Number series and their relationship with number sequences. Research of positive numerical series for	Number series and their relationship with number sequences. Cauchy criterion. Properties of numerical series. Positive number series and sufficient signs of their convergence: Dalambert's sign,

	convergence and sufficient signs of their convergence: Dalambert's sign, Cauchy's radical sian, Cauchy-Maclauren's integral sign; signs of Raabe and Gauss.	Cauchy's radical sign, Cauchy-McLauren's integral sign; signs of Raabe and Gauss. Tasks for **IWS**: p.6, No. 24;
25	**Lecture № 13.** Sign-alternating and sign-changing numerical series. Absolute and conditional convergence.	Sign-alternating number series. Leibniz's theorem (sign). Interchangeable rows. Absolute and conditional convergence. Permutation of members of sign-changing series. Riemann's theorem, Cauchy's theorem. Abel's identity, Dirichlet-Abel sign. Paradoxes associated with the permutation of members of alternating series. The concept of numerical series with complex terms. Tasks for **IWS**: p.6, No. 25;
26	**Practical lesson №13.** Sign-alternating and sign-changing numerical series. Research on absolute and conditional convergence. Leibniz's sign .	Sign-alternating and sign-changing numerical series. Research on absolute and conditional convergence. Leibniz's sign. Tasks for **IWS**: p.6, No. 26;
Topic 6. Power series. Taylor and Maclaurin series. Practical application of expanding functions into power series.		
27	**Lecture № 14**. Power series as a special case of functional series. Domain, interval, radius of convergence of a power series. Properties of convergent power series.	Definition of the area, interval and radius of convergence of power series. Abel's theorem. The Cauchy-Hadamard theorem. Properties of convergent power series. Terms of term-wise integration and differentiation of power series. Continuity of the sum of the power series. Calculation of sums of numerical series using power series. Abel's method. Tasks for **IWS**: p.6, No. 27;
28	**Practical lesson №14.** Research of power series for convergence: area, interval, radius of convergence of a power series. Properties of convergent power series. Calculation of sums of numerical series using power series. Abel's method.	Research on the convergence of power series: area, interval, radius of convergence of a power series. Properties of convergent power series. Calculation of sums of numerical series using power series. Abel's method. Tasks for **IWS**: p.6, No. 28;
29	**Lecture №18**. The technique of developing elementary functions in Taylor and McLaren series. Practical application of the development of functions in power series.	The technique of developing elementary functions in Taylor and McLaren series. Necessary and sufficient conditions for the development of functions in Taylor (McLauren) series. The simplest representations of power series in the complex domain. Practical application of the development of functions in power series. Tasks for **IWS**: p.6, No. 29;
30	**Practical lesson №18. On the first half-pair:** Technique of development of elementary functions in Taylor and McLaren series. Practical application of the development of functions in power series. Power series in the complex domain. **Within the framework of the second half-part**, **the third part of the MCW based on the material of Section III.**	The technique of developing elementary functions in Taylor and McLaren series. Practical application of the development of functions in power series. Power series in the complex domain. Tasks for **IWS**: p.6, No. 30;

6. Independent work of the student (IWS)

Mastering the educational material from the discipline "Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables " is based on self-preparation for classroom classes on theoretical and practical topics.

№ з/п	The name of the topic submitted for independent processing	Number of hours	Literature
1	Preparation for the lecture 1	1	[1-4], [6], [7], p.80–94
2	Preparation for practical lesson 1	0.5	[6,8,9],[7], p.80– 94
3	Preparation for the lecture 2	1	[1-4], [6], [7], p. 80– 94.
4	Preparation for practical lesson 2	0.5	[6,8,9], [7], p. 80–94.
5	Preparation for the lecture 3	1	[1-4], [6], [7], p. 80–94.
6	Preparation for practical lesson 3	0.5	[6,8,9], [7], p. 80–94
7	Preparation for the lecture 4	1	[1-4], [6], [7], p. 80–94.
8	Preparation for practical lesson 4	0.5	[6,8,9], [7], p. 80– 94.
9	Preparation for the lecture 5	1	[1-4], [6], [7], p. 80– 94.
10	Preparation for practical lesson 5	0.5	[6,8,9], [7], p. 80– 94.
11	Preparation for the lecture 6	1	[1-4], [6], [7], p.80– 94.
12	Preparation for practical lesson 6	0.5	[6,8,9], [7], p. 80– 94.
13	Preparation for the lecture 7	1	[2,5,6,9,10], [7], p.125– 139.
14	Preparation for practical lesson 7	0.5	[2,5,6,9,10], [7], p. 125– 139.
15	Preparation for the lecture 8	1	[2,5,6,9,10], [7], p.125– 139.
16	Preparation for practical lesson 8	0.5	[2,5,6,9,10], [7], p.125– 139.
17	Preparation for the lecture 9	1	[2,5,6,9,10], [7], p.125– 139.
18	Preparation for practical lesson 9	0.5	[2,5,6,9,10], [7], p.125– 139.
19	Preparation for the lecture 10	1	[2,5,6,9,10], [7], p.125– 139.
20	Preparation for practical lesson 10	0.5	[2,5,6,9,10], [7], p.125– 139.
21	Preparation for the lecture 11	1	[2,5,6,9,10], [7], p.125– 139.
22	Preparation for practical lesson 11	0.5	[2,5,6,9,10], [7], p.125– 139.
23	Preparation for the lecture 12	1	[2, 3, 5, 6, 8-10], [7], p.p. 95-109
24	Preparation for practical lesson 12	0.5	[2, 3, 5, 6, 8-10], [7], p.p. 95-109
25	Preparation for the lecture 13	1	[2, 3, 5, 6, 8-10], [7], p.p. 95-109
26	Preparation for practical lesson 13	0.5	[2, 3, 5, 6, 8-10],

			[7], p.p. 95-109
27	*Preparation for the lecture 14*	*1*	*[2, 3, 5, 6, 8-10], [7], p.p. 95-109*
28	*Preparation for practical lesson 14*	*0.5*	*[2, 3, 5, 6, 8-10], [7], p.p. 95-109*
29	*Preparation for the lecture 15*	*1*	*[2, 3, 5, 6, 8-10], [7], p.p. 95-109*
30	*Preparation for practical lesson 15*	*1*	*[6], [7], p. 95– 109.*
31	***Section III. Series.*** *Functional series (FS), region of convergence, uniform convergence. Dirichlet-Abel and Weierstrass signs. To master the concept of limit transition: a) under the sign of the integral and term-wise integration of the FS; b) under the sign of the derivative and term-wise differentiation of the FS. Analyze the proof of Dini's Theorem on the uniform convergence of a monotonic sequence of continuous functions to a continuous limit function. Functional properties of the sum of series, transition to the limit, integration and differentiation in functional series. Continuity conditions of the limit sum of a functional series.*	*4*	*[1-10], [13,14]*
32	*Preparation for MCW*	*13*	*[1-10], [13,14]*
33	*Preparation for IWS*	*20*	*[1-10], [13,14]*
34	*Preparation for the exam*	*30*	*[1-10], [13,14]*

Policy and control

7. Policy of academic discipline (educational component)

1. **General policy of teaching** the discipline *"Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables"* is aimed at independent performance of educational tasks, tasks of current and final control of learning results (for persons with special educational needs, this requirement is applied taking into account their individual needs and capabilities); mandatory correct reference to sources of information in the case of using other people's ideas, developments, statements, technologies; providing reliable information about the results of one's own educational (scientific, creative) activities, used research methods, technologies and sources of information,
2. **Visitation Policy**. In the normal course of study, attendance at both lectures and practical classes is a mandatory component of assessment. In long-term force majeure circumstances (military actions, pandemics, international internships), training can be conducted remotely. In this case, the absence of a classroom lesson does not involve the calculation of penalty points, since the student's final rating score is formed exclusively during the final examination. At the same time, independent performance of modular control tasks and defense of individual thematic tasks, as well as speeches (reports) at colloquiums and active work in practical classes will be evaluated during classroom classes.
3. **Policy on working out and redoing assessment control measures**. According to the regulation "Regulations on current, calendar and semester control of study results at Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf) every student has the right to make up for classes missed for a good reason and assessment control measures (hospital, mobility, etc.) at the expense of independent work.
4. **The procedure for contesting the results of assessment control measures**. According to the "Regulations on the resolution of conflict situations in the Igor Sikorsky Kyiv Polytechnic Institute" (https://osvita.kpi.ua/node/169) students have the right to challenge the results of the control measures with arguments, explaining which criterion they disagree with according to the assessment. A student may raise any issue relating to the assessment procedure and expect it to be dealt with in accordance with pre-defined procedures.
5. **Academic integrity**. The policy and principles of academic integrity are governed by the norms set forth in Chapter 3 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).

6. **Norms of ethical behavior**. The norms of ethical behavior of students and scientific and pedagogical workers are regulated by the provisions set forth in Chapter 2 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).
7. **Inclusive education**. Acquisition of knowledge and skills in the course of studying the discipline *"Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables"* is accessible to most people with special educational needs, except for those with severe visual impairments who cannot complete tasks using personal computers, laptops and/or other technical aids.
8. **Calendar control** is carried out in order to improve the quality of students' education and monitor the student's fulfillment of the syllabus requirements. Read more: Chapter 3 "Regulations on current, calendar and semester control of study results at the Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf).
9. **Studying in a foreign language**. In the process of mastering the lecture material and performing practical tasks in the discipline *"Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables"* students are recommended to refer to the English-language sources listed in the lists of basic and additional literature.
10. **Assignment of incentive and penalty points**. In accordance with the "Regulations on the system of evaluation of learning results at the Igor Sikorsky Kyiv Polytechnic Institute" the sum of all incentive points cannot exceed 10% of the rating scale (https://osvita.kpi.ua/node/37). The rules for assigning incentive and penalty points are as follows.

    Incentive points are awarded for: a) writing theses, articles, design of a new mathematical problem/technology as a scientific work for participation in a competition of student scientific works (on the subject of the academic discipline *"Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables"*) – up to 2 points; b) participation in international or all-Ukrainian events and competitions (on the subject of the academic discipline *"Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables"*) - up to 3 points.

    Penalty points are awarded for 1). Violation of the principles of academic integrity (non-independent performance of MCW and IWS, writing off during the exam): - 5 points for each violation (attempted plagiarism); multiple attempts at plagiarism are considered at the department meeting as force majeure with the adoption of strict measures against violators, including expulsion from the university; 2). delay in submission of MCW and IWS; the grade for each part of the semester individual IWS or CGW submitted a week later than the previously specified deadlines is reduced by 10%.

    Self-examination, preparation for practical classes, performance of individual tasks and control measures are carried out during independent work of students with the possibility of consulting with the teacher at the specified consultation time or by means of electronic correspondence (e-mail, messengers).

8. Types of control and rating system for evaluating learning outcomes (ELO)

Rating of the discipline *"Mathematical analysis. Part 2. Differential and integral calculus of functions of many variables"* consists of:
1) points for individual calculation and graphic work (IWS),
2) points for the integrated modular control work (MCW),
3) points for answering the exam,
4) incentive points,
5) penalty points.

RATING SYSTEM OF EVALUATION (ELO)

8.1 Points for the implementation and protection of an individual IWS.

During each semester, students perform one individual IWS, which is divided into all thematic sections.
The maximum number of points for a semester individual IWS: 20 points (total).
Points are awarded for:
- quality of execution (for individual IWS): 0-8 points;
- answer during the defense (for individual IWS): 0-8 points;
- timely submission of work for defense: 0-4 points.

Performance evaluation criteria:
8 points – the work is done qualitatively, in full;
6 points - the work is done qualitatively, in full, but has shortcomings;
3 points - the work is completed in full, but contains minor errors;
0 points – the work is incomplete or contains significant errors.
Criteria for evaluating the quality of the answer:
8 points – the answer is complete, well-argued;
6 points – the answer is generally correct, but has flaws or minor errors;
3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Criteria for evaluating the timeliness of work submission for defense:
4 points – the work is presented for defense no later than the specified deadline;
2 points – the work is submitted for defense 1 week later than the specified deadline;
0 points – the work is submitted for defense more than 1 week later than the specified deadline.
The maximum number of points for the implementation and defense of the semester individual IWS: 8+8+4=20 points (in total for three thematic sections).

8.2 Points for the completion of the semester integrated modular control work (MCW).

During the semester, students complete one semester-long integrated modular test, divided by thematic sections into four equal parts in terms of points and time; all tasks are written, including one theoretical and three practical.

The maximum total number of points for the integrated semester MCW is 30 points.

Criteria for evaluating written tasks of the integrated MCW:
30 points - the solution of the MCW tasks is absolutely (100%) correct;
27 points - the solution of the vast majority of tasks is correct, but in 10% of the tasks there are insignificant errors;
24 points – most tasks are solved correctly, but 20% of the tasks contain errors;
10-15 points - half of the tasks are solved correctly, but 50% of the tasks have significant errors;
5-6 points - the solution of 20% of the tasks is correct, but there are significant errors in 80% of the tasks;
1-3 points – the solution of 10% of tasks is correct, but 90% of tasks have significant errors;
0 points - there is no answer or the answer is 100% incorrect.

The semester component of the rating scale**: $R_S = R_{IWS} + R_{MCW}$ = 20+30 points = 50 points**.

8.3. Penalty points.

Penalty points are calculated for:
- academic dishonesty (plagiarism, non-independent performance of MCW, IWS, etc.) - 5 points for one attempt;
- delay of deadlines for submission of IWS, CGW; the grade for each part of the semester individual IWS or CGW submitted a week later than the specified deadlines is reduced by 10%.

8.4. Points for answers on the exam.

The examination ticket consists of 6 questions - 1 theoretical and 5 practical. The answer to a theoretical question is worth 10 points, and the answer to each practical question is worth 8 points.
Evaluation criteria for the theoretical question of the examination paper:
10 points – the answer is correct, complete, well-argued;
8-9 points – the answer is correct, detailed, but not very well argued;
6-7 points - in general, the answer is correct, but has flaws;
4-5 points – there are minor errors in the answer;
1-3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Evaluation criteria for the practical question of the examination work:
8 points – the answer is correct, the calculations are completed in full;
6-7 points - the answer is correct, but not very well supported by calculations;
5 points - in general, the answer is correct, but has flaws;
3-4 points – there are minor errors in the answer;
1-2 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.

The maximum number of points for an answer on the exam:
R_E = 10 points × 1 theoretical question + 8 points × 5 practical tasks = 50 points.

8.5. Calculation of the rating scale (R).

The semester component of the rating scale R_S= 50 points, it is defined as the sum of positive points received for the completion of the integral modular control work, for the completion and defense of the individual IWS and negative penalty points.
The examination component of the rating scale is equal to: R_E = 50 points.
The rating scale for the discipline is equal to: $R = R_S + R_E$ = 100 points.

8.6. Calendar control: is carried out twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements:

At the first certification (7-th week), the student receives "credited" if his current rating is at least 10 points (50% of the maximum number of points a student can receive before the first certification).
At the second certification (13-th week), the student receives "passed" if his current rating is at least 20 points (50% of the maximum number of points a student can receive before the second certification).

8.7. Conditions for admission to the exam and determining the grade.
A necessary condition for a student's admission to the exam is the completion and defense of all individual works and the student's semester rating of at least 60% of the R_S, i.e. at least 30 points. Otherwise, the student must do additional work and improve his rating.
The total rating of the R student is defined as the sum of the semester rating of the student R_S and the R_E points received on the exam. The grade is assigned according to the value of R according to the table. 1.

Table 1

Total rating R_D	**Grade**
95-100	excellent
85-94	very good
75-84	good
65-74	satisfactory
60-64	enough
$R_D \leq 59$	unsatisfactory
r_C < 30 or not performed (not protected) all types of work.	not allowed

8.8 Additional information on the discipline (educational component)
A copy of the typical exam tickets issued for each semester control is given in Appendix 1.

Working program of the academic discipline (syllabus):
Compiled by DcS., prof. Legeza V.P.
Approved by the department of software of computer systems
(protocol No. 11 from 30.04.2025)
Agreed by the Methodical Commission of the Faculty of Applied Mathematics
(protocol No. 11 from 23.05.2025)

Appendix 1

TYPICAL EXAMINATION TICKET
FROM THE DISCIPLINE "MATHEMATICAL ANALYSIS-2"
FOR SEMESTER CONTROL (SECOND SEMESTER)

<table>
<tr><td colspan="4">Igor Sikorsky Kyiv Polytechnic Institute</td></tr>
<tr>
<td>Educational and qualification level Bachelor Specialty F2 "Software Engineering"</td>
<td>Department of software of computer systems

2025 – 2026 education year</td>
<td>EXAMINATION TICKET No. 1 from the academic discipline

MATHEMATICAL ANALYSIS-2
Second semester</td>
<td>I approve
Chief Department of software of computer systems
____
(signature)
DcS., Assoc.prof.
E.S. Sulema

Prot. No. 11 dated 30.04.2025</td>
</tr>
<tr><td colspan="4">Examination theoretical questions</td></tr>
<tr><td colspan="4">1. Number series. Sufficient signs of convergence of positive numerical series. Cauchy's integral and radical sign (with proof). An example of the application of Cauchy's integral sign.</td></tr>
<tr><td colspan="4">Practical tasks of various types</td></tr>
<tr><td colspan="4">2. The function $f(x,y)$ is defined as follows:
$$f(x,y)=\begin{cases}\dfrac{x^2y}{x^2+y^2}, & x^2+y^2\neq 0;\\ 0, & x^2+y^2=0.\end{cases}$$
It is necessary to find out: 1). Is this function continuous? 2). Do its partial derivatives exist on the entire plane OXY ? 3). Are there points of the plane OXY at which the continuity of the partial derivatives of this function is broken? Justify the answers.</td></tr>
<tr><td colspan="4">3. Find the extrema of the function $z=x^4+y^4-2x^2+4xy-2y^2$ and set their type.</td></tr>
<tr><td colspan="4">4. Solve first-order DE using the method of variation of an arbitrary constant: $y'+\dfrac{y}{x}=x^2y^4$.</td></tr>
<tr><td colspan="4">5. Using the results of the theorem on superposition of solutions, find the partial and general solutions of the second-order DE: $y''-5y'+6y=12e^{-x}+18x^2-7$.</td></tr>
<tr><td colspan="4">6. Find the region of convergence of a power series with complex terms: $\sum_{n=0}^{\infty}\dfrac{(z+3-4i)^n}{(n+3)^2\,5^n}$.</td></tr>
</table>

Lecturer of the academic discipline, prof. ____________ V.P. Legeza