MATHEMATICAL ANALYSIS. Part 2. Functions of many variables, ordinary differential equations and series

Working program of the academic discipline (Syllabus)

Details of the academic discipline

Level of higher education	*First (undergraduate)*
Branch of knowledge	*12 Information technologies*
Specialty	*121 Software engineering*
Educational program	*Software Engineering of Multimedia and Information Retrieval Systems*
Discipline status	*Normative*
Form of education	*Daytime*
Year of training, semester	*First year of training, second semester*
Scope of the discipline	*Lectures: 36 hours, practical classes: 36 hours, independent work: 78 hours.*
Semester control	*Exam, modular control work, calculation and graphic work, calendar control*
Timetable	*According to the schedule for the spring semester of the current academic year (rozklad.kpi.ua)*
Language of teaching	*English*
Information about the course leader / teachers	*Lecturer: doctor of science, professor, Legeza Viktor Petrovych* *legeza@pzks.fpm.kpi.ua* *Practical training: Ph.D., associate professor, Oleksandr Mykhailovych Neshchadym, om.neshchadym@gmail.com*
Placement of the course	*second semester: https://classroom.google.com/w/MjY4ODc3OTMyOTM5/t/all*

Program of study discipline:

1. Description of the educational discipline, its purpose, subject of study and learning outcomes

Study of the discipline “Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series” allows students to develop the competencies necessary for building mathematical models and algorithms in the process of research and solving practical problems of natural science and information technologies.

The purpose of studying the discipline “Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series” is the formation of a higher education student's abilities for abstract thinking, independent analysis and synthesis of complex multidimensional systems, as well as the ability to use the acquired fundamental knowledge at the stages of posing a problem in mathematical and symbolic form, followed by its algorithmization and development of modern software.

The subject of the discipline is “Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series” are methods, techniques and technologies of mathematical analysis and its additional sections, which make up the theoretical justification and mathematical support of the process of solving a wide range of problems belonging to the field of knowledge 12 "Information technologies".

Study of the discipline “Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series” will contribute to the formation of the following **general (GC)** and **professional competences** (PC):

GC01 Ability to abstract thinking, analysis and synthesis.

GC02 Ability to apply knowledge in practical situations.

GC06 Ability to search, process and analyze information from various sources.

PC15 Ability to apply fundamental and interdisciplinary knowledge to build advanced retrieval algorithms.

PC16 Ability to develop algorithms for implementing statistical data analysis methods.

PC18 Ability to develop methods for mathematical problems numerical solutions using software.

PC20 Ability to apply the acquired fundamental mathematical knowledge to develop calculation methods in the multimedia and information retrieval systems creation.

Study of the discipline “Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series” will contribute to the formation of the following **program learning outcomes** (PLO) for students under the EP:

PLO05 To know and apply relevant mathematical concepts, domain methods, system and object-oriented analysis and mathematical modeling for software development.

PLO11 To select initial data for design, guided by formal methods of describing requirements and modeling.

PLO25 To know and to be able to use fundamental mathematical tools in the algorithms construction and modern software development.

PLO26 To be able to develop and use methods and algorithms for the mathematical problems approximate solution during the multimedia and information retrieval systems design.

PLO27 To be able to use statistical data analysis methods.

PLO28 To know the mathematical and algorithmic basics of computer graphics and to be able to apply them to develop multimedia software.

2. Pre-requisites and post-requisites of the discipline (place in the structural and logical scheme of training according to the relevant educational program)

Successful study of the discipline "Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series" should be provided as part of the thoroughly mastered educational material of the discipline "Mathematical analysis. Part 1. Differential and integral calculus of functions of one variable", as well as the discipline "Linear algebra and analytical geometry".

Received during mastering the discipline "Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series" theoretical knowledge and practical skills are necessary for studying the disciplines "Probability theory", "Mathematical support of multimedia and information retrieval systems", "Physical foundations of multimedia systems", "Algorithmic support of multimedia and information retrieval systems" of the curriculum of bachelor's training in the specialty 121 Software engineering, as well as the disciplines "Operations research and mathematical programming" and "Information retrieval systems and services" of the curriculum of the master of science training under the EP "Software engineering of multimedia and information retrieval systems".

3. Content of the academic discipline.

Discipline "Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series" involves the study of the following topics:

Topic 1. Boundary, continuity, differentiability of FMV.

Topic 2. Application of partial derivatives.

Topic 3. Differential equations of the first order.

Topic 4. Differential equations of higher orders. Normal DR systems.

Topic 5. Number series. Signs of convergence.

Topic 6. Functional and power series.

Modular control work (MCW).

Exam

4. Educational materials and resources

Basic literature

1. G. M. Fikhtengol'ts, The Fundamentals of Mathematical Analysis. (International Series of Monographs in Pure and Applied Mathematics, Vol. 72/73) Vol. 1: 491 p. Oxford/London/Edinburg/New York/Paris/Frankfurt 1965.
2. G. M. Fikhtengol'ts, The Fundamentals of Mathematical Analysis. (International Series of Monographs in Pure and Applied Mathematics, Vol. 72/73) Vol. 2: 518 p. Oxford/London/Edinburg/New York/Paris/Frankfurt 1965.
3. S. M. Nikolsky. Course in Mathematical Analysis. Vol. 1: Mir Publishers, 1977. 460 p.
4. V.A.Ilyin and E.G.Pozyak. Fundamentals of mathematical analysis. Part 1. Mir Publishers, 1982, 637 p. https://www.biblio.com/book/fundamentals-mathematical-analysis-ilyin-v-e/d/734370748.
5. V.A. Ilyin and E.G. Pozyak. Fundamentals of mathematical analysis. Part 2 Mir Publishers, 1982, (438 p.). https://www.biblio.com/book/fundamentals-mathematical-analysis-ilyin-v-e/d/734370748.
6. B.P. Demidovich. Problems in Mathematical Analysis. Gordon & B., 1969, 496 p.
7. V.P. Legeza. Mathematical analysis: a collection of problems. - Kyiv, KPI, Polytechnic Publishing House, 2018. - 240 p.
8. Dennis G. Zill. First Course in Differential Equations with Modelling Applications. Loyola Marymount University, Tenth Edition, 2012. https://eduguidehome.files.wordpress.com/2019/02/a-first-course-in-differential-equations-10th-edition-by-dennis-g-zill.pdf

Additional literature.

9. S.V.Budak, B.M.Fomin. Multiple Integrals, Field Theory and Series. An Advanced Course in Higher Mathematics. Mir Publishers; First printing edition, 1973. – 608 p. https://books.google.com.ua/books/about/Multiple_Integrals_Field_Theory_and_Seri.html?id=XKQNAQAAIAAJ&redir_esc=y
10. Y.B.Zel`dovich, A.D.Myshkis. Elements of Applied Mathematics. Mir Publishers, 1976, 656 p.
11. R.Courant. Differential and Integral Calculus. Vol. 2. Ishi press international, 2010, 682 p.
12. G.N.Berman. A problem book in mathematical analysis. MTG Learning Media (P) Ltd., New Delhi/Gurgaon, 2017, 490 p.
13. N.Piskunov. Differential and Integral calculus. Vol. 1,2, Mir Publishers, 1969. 895 p. https://mmsallaboutmetallurgy.com/wp-content/uploads/2019/01/differential-and-integral-calculus-by-piskunov.pdf.
14. M.Tenenbaum, H.Pollard. Ordinary differential equations. Dover Publications Inc., 1985, 818 p.

5. Methods of mastering an educational discipline (educational component)

№ з/п	Educational type occupation	Description of the training session
	Section I. Differential calculus FMV. ***Topic 1. Boundary, continuity, differentiability of FMV***	
1	**Lecture №1**. Basic concepts and definitions of functions of many variables; Euclidean plane and Euclidean space E_n. Level lines and surfaces.	The concept of FMV. Coordinate and Euclidean space. Graphs of a function of two variables. Level lines and surfaces. Metric space and its axioms. n – dimensional sphere and n – dimensional parallelepiped. The concept of a ε – neighborhood of point in space E_n. Open, closed, bounded and connected sets. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
2	**Practical lesson №1**. Graphs of a function of two variables. Construction of isolines and	Graphing a function of two variables. Construction of isolines and isosurfaces. The limit of a sequence of points in Euclidean space. The concept of a fundamental sequence of points in Euclidean

	isosurfaces. The limit of a sequence of points in Euclidean space. The technique of calculating the borders of the FMV. Operations over the borders of the FMV.	space. The technique of calculating the borders of the FMV. Operations over the borders of the FMV. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
3	**Lecture №2**. Definition of FMV and limits of the sequence of points $\{M_k\}$ of Euclidean space. E_n	Definition of FMV. The limit of a sequence of points in Euclidean space. Lemma about coordinate convergence of a sequence of points in Euclidean space. The concept of a fundamental sequence of points in Euclidean space. Cauchy criterion. Bolzano-Cauchy theorem for FMV. The concept and two definitions of the FMV boundary according to Heine and Cauchy. Theorem on arithmetic operations on the boundaries of FMV. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
4	**Practical lesson №2.** Operations on infinitesimal FMV. Main and repeated limits. Research of FMV for continuity. Study of FMV for uniform continuity according to Cantor.	Operations on infinitesimal FMV. Main and repeated limits. Research of FMV for continuity. Study of FMV for uniform continuity according to Cantor. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
5	**Lecture №3.** Infinitesimal FMV. Cauchy criterion. Repeated limits. Continuity of FMV. Properties of continuous functions.	Definition of infinitesimal FMV. Necessary and sufficient conditions for the existence of the FMV limit. The concept of the main and repeated limits of FMV. Sufficient conditions for the existence and equality of the repeated limits of the FMV. Concept and three definitions of FMV continuity. Properties of continuous functions. Two Theorems of Weierstrass. Uniform continuity. Cantor's theorem. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
6	**Practical lesson №3.** The technique of finding partial derivatives of FMV. Research of FMV on differentiability. Using the first differential for approximate calculations.	The technique of finding partial derivatives of FMV. Research of FMV on differentiability. Using the first differential for approximate calculations. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
7	**Lecture №4.** Partial derivatives of FMV. The concept of differentiability of FMV. The first differential and its application to approximate calculations.	Definition of partial derivatives. Partial derivatives of higher orders. Schwartz's theorem. Definition of differentiability of FMV, necessary and sufficient conditions for differentiability of function of two variables. The fundamental difference between the differentiation of FMV and the differentiation of FOV. The first differential of a function of two variables, its connection with the existence of partial derivatives and application to approximate calculations. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
8	**Practical lesson №4.** The technique of finding differentials of higher orders. The derivative of the composite FMV. Invariance of the form of the complete differential. The technique of differentiating implicit functions. Taylor's formula for a function of two variables.	The technique of finding differentials of higher orders. The derivative of the composite FMV. Invariance of the form of the complete differential. The technique of differentiation of implicit functions. Taylor's formula for a function of two variables. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
9	**Lecture №5.** Differentials of higher orders. The derivative of the composite FMV. Invariance of the form of the complete differential. Differentiation of implicit functions. Taylor's formula for a function of two variables.	Differentials of higher orders. The formula for the differential of $n-$th order. The derivative of the composite FMV. Full derivative. Invariance of the form of the complete differential. Theorems on the existence of implicit functions of one and two variables. Differentiation of implicit functions. Taylor and Maclauren's formula for a function of two variables. The residual term of Taylor's formula in Lagrange form.. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
10	**Practical lesson №5.** Application of partial derivatives: tangent plane and normal to the surface. Derivative in direction. Gradient and its properties.	Tangent plane and normal to the surface. Derivative in direction. The physical content of the gradient and its properties. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94i.
Topic 2. Application of partial derivatives.		

11	**Lecture №6.** Tangent plane and normal to the surface. Derivative in direction. Gradient and its properties.	Definition of the tangent plane and the normal to the surface. The geometric meaning of the differential of a function of two variables. Definition of a scalar field. Types of scalar fields. Derivative in direction, its physical meaning. Concept of gradient, its properties. The connection of the gradient with the derivative in the direction.Tasks for **IWS**: [1-4], [7], p.p. 80-94
12	**Practical lesson №6.** Local extremum of FMV. The largest and smallest value of a continuous function in a closed bounded region. Conditional extremum. Lagrange function and multipliers. The method of least squares. Regression.	Local extremum of FMV. The largest and smallest value of a continuous function in a closed bounded region. Conditional extremum. Lagrange function and multipliers. The method of least squares. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
13	**Lecture №7.** Local extremum of FMV. The largest and smallest value of a continuous function in a closed bounded region. Conditional extremum. Lagrange function and multipliers. The method of least squares.	Local extremum of a function of two variables. Stationary and critical points of the function. Necessary and sufficient conditions for the existence of a local extremum. A rule for investigating a function of two variables for a local extremum. Concept of quadratic forms. Sylvester criterion. The largest and smallest value of a continuous function in a closed and bounded region. Conditional extremum. Elm equation. Lagrange function and multipliers. The method of least squares. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
14	**Practical lesson №7.** ***On the first half-pair:*** DE of the first order and methods of their integration: homogeneous, linear, Bernoulli and Riccati equations. ***In the second half:*** the first part of the MCW on topics #1-2.	***On the first half-pair:*** DE of the first order and methods of their integration: homogeneous, linear, Bernoulli and Riccati equations. ***In the second half:*** the first part of the MCW on topics #1-2. Tasks for **IWS**: [1-4], [6], [7], p.p. 80-94
	Section II. Ordinary differential equations (ODEs). ***Topic 3. Differential equations of the first order***	
15	**Lecture №8**. Ordinary differential equations (ODEs) of the first order. Cauchy's problem. Theorems of Cauchy and Picard. Isoclines. Differential equations with separable variables.	Ordinary DEs of the first order: general concepts and definitions. Normal form of DE, DE in differential form. Solution of DE, integral curves. Formulation of the Cauchy problem, initial conditions. Cauchy's theorem on the existence and unity of the first-order DE solution. Special interchanges DE. The concept of general and partial solutions of DE. Picard's theorem. Lipshitz conditions. Picard's method of successive approximations. The geometric content of DE of the first order. Field of directions, isoclines. The concept of integration of DE in quadrature. Differential equations with separable variables. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
16	**Practical lesson №8.** DE in full differentials, the technique of their integration. Integrating factor.	DE in full differentials, the technique of their integration. Using the integrating factor to solve the first-order DE. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
17	**Lecture №9**. Other types of DE of the first order.	Homogeneous functions of the th dimension. Homogeneous functions of zero dimension. Homogeneous DEs of the first order. The method of their integration. DE, which are reduced to homogeneous. Linear DEs of the first order. Bernoulli substitution. DE, which are reduced to linear. Bernoulli's equation. The Riccati equation. Cases in which the Riccati equation is integrated in quadratures. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
18	**Practical lesson №9.** DE of the $n-$th order, which are integrated in quadrature. Differential equations of higher orders that allow integration in quadratures.	DE of the $n-$th order, which are integrated in quadrature. Differential equations of higher orders that allow integration in quadratures. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
19	**Lecture №10**. DE in full differentials, the technique of its integration. Integrating factor. Clairot and Lagrange equations (IWS).	Definition of DE in complete differentials, technique of its solution. The concept of an integrating factor; the technique of finding it if $\mu = \mu(x)$ or $\mu = \mu(y)$. Clairo's equation: definition and method of its integration. The concept of general and special solutions of the Clerot equation. The family of straight lines as a

		geometric image of the general solution of the Clerot equation. Lagrange's equation: definition and method of its integration. The concept of general and special solutions of the Lagrange equation. Parametric representation of the solution of the Lagrange equation. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
20	**Practical lesson №10.** The technique of solving linear DEs of the second order Characteristic equation. Fundamental system of solutions. Vronsky's determinant. Ostrogradsky-Liouville formula. The structure of the general junction DE.	Linear DEs of the second order Characteristic equation. Fundamental system of solutions. Vronsky's determinant. Ostrogradsky-Liouville formula. The structure of the general junction DE. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
Topic 4. Differential equations of higher orders. Normal DE systems		
21	**Lecture №11**. Differential equations of higher orders that allow integration in quadratures.	DR of higher orders: basic concepts and definitions. Cauchy's problem, initial conditions. General and partial solutions of DE of higher orders, geometric interpretation of the general solution using integral curves. Cauchy's theorem on the existence and uniqueness of the solution of the Cauchy problem. DE of the $n-$th order, which are integrated in quadrature. Some types of equations of higher orders that allow reduction of order. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
22	**Practical lesson №11.** Differential equations of higher orders that allow integration in quadratures. Some types of equations of higher orders that allow reduction of order.	Differential equations of higher orders that allow integration in quadratures. Some types of equations of higher orders that allow reduction of order Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
23	**Lecture №12.** Linear DEs of the second order (general case). The method of variation of arbitrary constants for the second-order LIDE.	Linear homogeneous DE of the second order (LHDE). Theorem on the linear combination of its two solutions. The concept of linear dependence (independence) of two functions on a given interval. Vronsky's determinant. Ostrogradsky-Liouville formula. The fundamental system of LHDE solutions. Criterion of linear independence of second-order LHDE solutions. Theorem on the structure of the general solution of the second order LHDE. A lemma on the restoration of the LHDE according to a given fundamental system of its solutions. Linear inhomogeneous DE of the second order (LIDE). The general solution of its homogeneous equation and the partial solution of the inhomogeneous equation. Theorem on the structure of the general solution of the second-order LIDE. The method of variation of arbitrary constants for the second-order LIDE. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
24	**Practical lesson №12.** Linear DEs of the second order. Method of variation of arbitrary constants for the second-order LIDE.	Linear DEs of the second order Method of variation of arbitrary constants for the second-order LIDE. The general solution of its homogeneous equation and the partial solution of the inhomogeneous equation. The structure of the second-order general solution of the LIDE. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
25	**Lecture №13.** Linear DEs of the second order with constant coefficients.	LHDE of the second order with constant coefficients. Definition and construction of the characteristic equation. Three cases of the roots of the characteristic equation, the construction of the corresponding general solutions of the second order LHDE for them. LIDE of the second order with constant coefficients. Designation of the LIDE with a special right part. The concept of a quasi-polynomial. The method of selecting a separate partial solution of the second-order LIDE under the condition that the right-hand side of the LIDE contains quasipolynomials of the form $f(x)=e^{\alpha x}P_n(x)$ or $f(x)=e^{\alpha x}\left[P_n(x)\cos(\beta x)+R_m(x)\sin(\beta x)\right]$. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124

26	**Practical lesson №13.** LHDE of the second order with constant coefficients. Construction of the characteristic equation. The technique of solving the LIDE with a special right-hand part. The concept of a quasi-polynomial.	LHDE of the second order with constant coefficients. Definition and construction of the characteristic equation. The technique of solving the LIDE with a special right-hand part. The concept of a quasipolynomial of the form: $f(x)=e^{\alpha x}P_n(x)$ or $f(x)=e^{\alpha x}\left[P_n(x)\cos(\beta x)+R_m(x)\sin(\beta x)\right]$. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
27	**Lecture №14.** Normal systems of differential equations. Reduction of any DE $n-$th order to a normal DE system. Reduction of the normal DE system to one DE of the $n-$th order. Method of variation of arbitrary constants**. (IWS).**	Normal DE systems: general concepts and definitions. Reduction of any DE th order to a normal DE system. Reduction of the normal DE system to one DE of the th order. Cauchy's theorem on the existence and uniqueness of the solution of the normal system DE. Systems of linear homogeneous equations (LHDE). Concept of derivative and integral of a matrix. Properties of solutions of the normal DE system. The fundamental system of solutions of the DE system. Construction of the general solution of the LHDE system with constant coefficients. Characteristic equation for the LHDE system with constant coefficients. LIDE systems with fixed coefficients. Method of variation of arbitrary constants. The method of selecting a separate partial solution of the LIDE system with constant coefficients. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
28	**Practical lesson №14.** Normal systems of differential equations. Reduction of any DE of $n-$th order to a normal system of DE. Reduction of the normal DE system to one DE of the $n-$th order. Method of variation of arbitrary constants. (**IWS**). **Within the framework of the current pair - the second part of the MCW according to section II.**	Normal systems of differential equations. The technique of reduction of any DE of $n-$th order to a normal system of DE. The technique of reduction of the normal DE system to one DE of the $n-$th order. Tasks for **IWS**: [8, 11-14], [7], p.p. 110-124
	Section III. Series ***Topic 5. Number series. Signs of convergence.***	
29	**Lecture № 15.** Number series and their relationship with number sequences. Cauchy criterion. Properties of numerical series. Positive numerical series and sufficient signs of their convergence.	Number series: basic concepts and definitions. Partial sum and the concept of convergence of numerical series. The study of numerical series as a new form of studying the properties of numerical sequences. The Cauchy criterion for the convergence of a numerical series. The simplest properties of number series. Remainder of a number series. A necessary condition for the convergence of the numerical series. A sufficient condition for the divergence of a number series. Sufficient signs of convergence of positive number series: sign of comparison by inequality and limit sign; Dalambert's sign, Cauchy's radical sign, Cauchy-McLauren's integral sign; signs of Raabe and Gauss. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109
30	**Practical lesson №15.** Number series and their relationship with number sequences. Research of positive numerical series for convergence and sufficient signs of their convergence: Dalambert's sign, Cauchy's radical sian, Cauchy-Maclauren's integral sign; signs of Raabe and Gauss.	Number series and their relationship with number sequences. Cauchy criterion. Properties of numerical series. Positive number series and sufficient signs of their convergence: Dalambert's sign, Cauchy's radical sign, Cauchy-McLauren's integral sign; signs of Raabe and Gauss. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109
31	**Lecture № 16.** Sign-alternating and sign-changing numerical series. Absolute and conditional convergence.	Sign-alternating number series. Leibniz's theorem (sign). Interchangeable rows. Absolute and conditional convergence. Permutation of members of sign-changing series. Riemann's theorem, Cauchy's theorem. Abel's identity, Dirichlet-Abel sign. Paradoxes associated with the permutation of members of alternating series. The concept of numerical series with complex terms. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109

32	**Practical lesson №16.** Sign-alternating and sign-changing numerical series. Research on absolute and conditional convergence. Leibniz's sign .	Sign-alternating and sign-changing numerical series. Research on absolute and conditional convergence. Leibniz's sign. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109
Тема 6. Функціональні та степеневі ряди.		
33	**Lecture № 17**. Power series: domain, interval, radius of convergence of a power series. Properties of convergent power series.	Definition of the area, interval and radius of convergence of power series. Abel's theorem. The Cauchy-Hadamard theorem. Properties of convergent power series. Terms of term-wise integration and differentiation of power series. Continuity of the sum of the power series. Calculation of sums of numerical series using power series. Abel's method. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109
34	**Practical lesson №17.** Research of power series for convergence: area, interval, radius of convergence of a power series. Properties of convergent power series. Calculation of sums of numerical series using power series. Abel's method.	Research on the convergence of power series: area, interval, radius of convergence of a power series. Properties of convergent power series. Calculation of sums of numerical series using power series. Abel's method. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109
35	**Lecture №18**. The technique of developing elementary functions in Taylor and McLaren series. Practical application of the development of functions in power series.	The technique of developing elementary functions in Taylor and McLaren series. Necessary and sufficient conditions for the development of functions in Taylor (McLauren) series. The simplest representations of power series in the complex domain. Practical application of the development of functions in power series. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109
36	**Practical lesson №18. On the first half-pair:** Technique of development of elementary functions in Taylor and McLaren series. Practical application of the development of functions in power series. Power series in the complex domain. **Within the framework of the second half-part**, **the third part of the MCW based on the material of Section III.**	The technique of developing elementary functions in Taylor and McLaren series. Practical application of the development of functions in power series. Power series in the complex domain. Tasks for **IWS**: [2, 3, 5, 6, 9], [7], p.p. 95-109

6. Independent work of the student (IWS)

Mastering the educational material from the discipline "Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series" is based on self-preparation for classroom classes on theoretical and practical topics.

№ з/п	*The name of the topic submitted for independent processing*	*Number of hours*	*Literature*
1	*Preparation for the lecture 1.*	*1*	*[1-4], [6], [7], p.80–94.*
2	*Preparation for practical lesson 1*	*0.5*	*[6], [7], p. 80– 94.*
3	*Preparation for the lecture 2.*	*1*	*[1-4], [6], [7], p. 80–94.*
4	*Preparation for practical lesson 2*	*0.5*	*[6], [7], p. 80– 94.*
5	*Preparation for the lecture 3*	*1*	*[1-4], [6], [7], p. 80–94.*
6	*Preparation for practical lesson 3*	*0.5*	*[6], [7], p. 80– 94*

7	*Preparation for the lecture 4*	*1*	*[1-4], [6], [7], p. 80–94.*
8	*Preparation for practical lesson 4*	*0.5*	*[6], [7], p. 80– 94.*
9	*Preparation for the lecture 5*	*1*	*[1-4], [6], [7], p. 80–94.*
10	*Preparation for practical lesson 5*	*0.5*	*[6], [7], p. 80– 94.*
11	*Preparation for the lecture 6*	*1*	*[1-4], [6], [7], p.80–94.*
12	*Preparation for practical lesson 6*	*0.5*	*[6], [7], p. 80– 94.*
13	*Preparation for the lecture 7*	*1*	*[1-4], [6], [7], p.80–94.*
14	*Preparation for practical lesson 7*	*0.5*	*[6], [7], p. 110– 124.*
15	*Preparation for the lecture 8*	*1*	*[8, 11-14], [6], [7], p. 110– 124*
16	*Preparation for practical lesson 8*	*0.5*	*[6], [7], p. 110– 124*
17	*Preparation for the lecture 9*	*1*	*[8, 11-14], [6], [7], p. 110– 124.*
18	*Preparation for practical lesson 9*	*0.5*	*[6], [7], p. 110– 124*
19	*Preparation for the lecture 10*	*1*	*[8, 11-14], [6], [7], p. 110– 124*
20	*Preparation for practical lesson 10*	*0.5*	*[6], [7], p. 110– 124*
21	*Preparation for the lecture 11*	*1*	*[8, 11-14], [6], [7], p. 110– 124*
22	*Preparation for practical lesson 11*	*0.5*	*[6], [7], p. 110– 124*
23	*Preparation for the lecture 12*	*1*	*[8, 11-13], [6], [7], p. 110– 124*
24	*Preparation for practical lesson 12*	*0.5*	*[6], [7], p. 110– 124*
25	*Preparation for the lecture 13*	*1*	*[8, 11-14], [6], [7], p. 110– 124*
26	*Preparation for practical lesson 13*	*0.5*	*[6], [7], p. 110– 124*
27	*Preparation for the lecture 14*	*1*	*[8, 11-14], [6], [7], p. 110– 124*
28	*Preparation for practical lesson 14*	*0.5*	*[6], [7], p. 110– 124*
29	*Preparation for the lecture 15*	*1*	*[2, 3, 5, 6, 9], [6], [7], p. 95– 109.*
30	*Preparation for practical lesson 15*	*0.5*	*[6], [7], p. 95– 109.*
31	*Preparation for the lecture 16*	*1*	*[2, 3, 5, 6, 9], [6], [7], p. 95–109.*
32	*Preparation for practical lesson 16*	*0.5*	*[6], [7], p. 95– 109.*
33	*Preparation for the lecture 17*	*1*	*[2, 3, 5, 6, 9], [6], [7], p. 95 – 109.*
34	*Preparation for practical lesson 17*	*0.5*	*[6], [7], p. 95 – 109.*
35	*Preparation for the lecture 18*	*1*	*[2, 3, 5, 6, 9], [6], [7], p. 95– 109*
36	*Preparation for practical lesson 18*	*0.5*	*[6], [7], p. 95 – 109.*
37	***Section II. Ordinary differential equations.*** *To master the technique of solving DE Clairot and Lagrange. Find out how*	*2*	*[8, 11-14]*

	the special solutions of these DEs differ from the general ones.		
38	***Section II. Ordinary differential equations.*** *To master the technique of solving systems of n first-order linear DEs with constant coefficients. The method of reducing a system of n first-order linear DEs to one n-th-order linear DE with constant coefficients.*	*2*	*[8, 11-14]*
39	***Section III. Numerical, functional and power series.*** *Functional series (FS), region of convergence, uniform convergence. Dirichlet-Abel and Weierstrass signs. To master the concept of limit transition: a) under the sign of the integral and term-wise integration of the FS; b) under the sign of the derivative and term-wise differentiation of the FS. Analyze the proof of Dini's Theorem on the uniform convergence of a monotonic sequence of continuous functions to a continuous limit function. Functional properties of the sum of series, transition to the limit, integration and differentiation in functional series. Continuity conditions of the limit sum of a functional series.*	*4*	*[8, 11-14]*
40	*Preparation for MCW*	*5*	*[1-9], [11-14]*
41	*Preparation for IWS*	*8*	*[1-9], [11-14]*
42	*Preparation for the exam*	*30*	*[1-9], [11-14]*

Policy and control

7. Policy of academic discipline (educational component)

1. **General policy of teaching** the discipline *"Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series"* is aimed at independent performance of educational tasks, tasks of current and final control of learning results (for persons with special educational needs, this requirement is applied taking into account their individual needs and capabilities); mandatory correct reference to sources of information in the case of using other people's ideas, developments, statements, technologies; providing reliable information about the results of one's own educational (scientific, creative) activities, used research methods, technologies and sources of information,
2. **Visitation Policy**. In the normal course of study, attendance at both lectures and practical classes is a mandatory component of assessment. In long-term force majeure circumstances (military actions, pandemics, international internships), training can be conducted remotely. In this case, the absence of a classroom lesson does not involve the calculation of penalty points, since the student's final rating score is formed exclusively during the final examination. At the same time, independent performance of modular control tasks and defense of individual thematic tasks, as well as speeches (reports) at colloquiums and active work in practical classes will be evaluated during classroom classes.
3. **Policy on working out and redoing assessment control measures**. According to the regulation "Regulations on current, calendar and semester control of study results at Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf) every student has the right to make up for classes missed for a good reason and assessment control measures (hospital, mobility, etc.) at the expense of independent work.
4. **The procedure for contesting the results of assessment control measures**. According to the "Regulations on the resolution of conflict situations in the Igor Sikorsky Kyiv Polytechnic Institute" (https://osvita.kpi.ua/node/169) students have the right to challenge the results of the control measures with arguments, explaining which criterion they disagree with according to the assessment. A student may raise any issue relating to the assessment procedure and expect it to be dealt with in accordance with pre-defined procedures.
5. **Academic integrity**. The policy and principles of academic integrity are governed by the norms set forth in Chapter 3 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).
6. **Norms of ethical behavior**. The norms of ethical behavior of students and scientific and pedagogical workers are regulated by the provisions set forth in Chapter 2 of the Code of Honor of the Igor Sikorsky Kyiv Polytechnic Institute (https://kpi.ua/code).

7. **Inclusive education**. Acquisition of knowledge and skills in the course of studying the discipline *"Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series"* is accessible to most people with special educational needs, except for those with severe visual impairments who cannot complete tasks using personal computers, laptops and/or other technical aids.
8. **Calendar control** is carried out in order to improve the quality of students' education and monitor the student's fulfillment of the syllabus requirements. Read more: Chapter 3 "Regulations on current, calendar and semester control of study results at the Igor Sikorsky Kyiv Polytechnic Institute" (https://kpi.ua/files/n3277.pdf).
9. **Studying in a foreign language**. In the process of mastering the lecture material and performing practical tasks in the discipline *"Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series"* students are recommended to refer to the English-language sources listed in the lists of basic and additional literature.
10. **Assignment of incentive and penalty points**. In accordance with the "Regulations on the system of evaluation of learning results at the Igor Sikorsky Kyiv Polytechnic Institute" the sum of all incentive points cannot exceed 10% of the rating scale (https://osvita.kpi.ua/node/37). The rules for assigning incentive and penalty points are as follows.
    Incentive points are awarded for: a) writing theses, articles, design of a new mathematical problem/technology as a scientific work for participation in a competition of student scientific works (on the subject of the academic discipline *"Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series"*) – up to 2 points; b) participation in international or all-Ukrainian events and competitions (on the subject of the academic discipline *"Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series"*) - up to 3 points.
    Penalty points are awarded for violations of the principles of academic integrity (non-independent performance of MCW and IWS, writing off during the exam): - 5 points for each violation (attempted plagiarism).
    Self-examination, preparation for practical classes, performance of individual tasks and control measures are carried out during independent work of students with the possibility of consulting with the teacher at the specified consultation time or by means of electronic correspondence (e-mail, messengers).

8. Types of control and rating system for evaluating learning outcomes (ELO)

Rating of the discipline *"Mathematical analysis. Part 2. Functions of many variables, ordinary differential equations and series"* consists of:
1) points for individual calculation and graphic work (IWS),
2) points for the integrated modular control work (MCW),
3) points for answering the exam,
4) incentive points,
5) penalty points.

RATING SYSTEM OF EVALUATION (ELO)

8.1 Points for the implementation and protection of an individual IWS.

During each semester, students perform one individual IWS, which is divided into all thematic sections.
The maximum number of points for a semester individual IWS: 20 points (total).
Points are awarded for:
- quality of execution (for individual IWS): 0-8 points;
- answer during the defense (for individual IWS): 0-8 points;
- timely submission of work for defense: 0-4 points.

Performance evaluation criteria:
8 points – the work is done qualitatively, in full;
6 points - the work is done qualitatively, in full, but has shortcomings;
3 points - the work is completed in full, but contains minor errors;
0 points – the work is incomplete or contains significant errors.
Criteria for evaluating the quality of the answer:
8 points – the answer is complete, well-argued;

6 points – the answer is generally correct, but has flaws or minor errors;
3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Criteria for evaluating the timeliness of work submission for defense:
4 points – the work is presented for defense no later than the specified deadline;
2 points – the work is submitted for defense 1 week later than the specified deadline;
0 points – the work is submitted for defense more than 1 week later than the specified deadline.
The maximum number of points for the implementation and defense of the semester individual IWS: 8+8+4=20 points (in total for three thematic sections).

8.2 Points for the completion of the semester integrated modular control work (MCW).

During the semester, students complete one semester-long integrated modular test, divided by thematic sections into four equal parts in terms of points and time; all tasks are written, including one theoretical and three practical.

The maximum total number of points for the integrated semester MCW is 30 points.

Criteria for evaluating written tasks of the integrated MCW:
30 points - the solution of the MCW tasks is absolutely (100%) correct;
27 points - the solution of the vast majority of tasks is correct, but in 10% of the tasks there are insignificant errors;
24 points – most tasks are solved correctly, but 20% of the tasks contain errors;
10-15 points - half of the tasks are solved correctly, but 50% of the tasks have significant errors;
5-6 points - the solution of 20% of the tasks is correct, but there are significant errors in 80% of the tasks;
1-3 points – the solution of 10% of tasks is correct, but 90% of tasks have significant errors;
0 points - there is no answer or the answer is 100% incorrect.

The semester component of the rating scale**: $R_S = R_{IWS} + R_{MCW}$ = 20+30 points = 50 points**.

8.3. Penalty points.

Penalty points are calculated for:
- academic dishonesty (plagiarism, non-independent performance of MCW, IWS, etc.) - 5 points for one attempt.

8.4. Points for answers on the exam.

The examination ticket consists of 6 questions - 1 theoretical and 5 practical. The answer to a theoretical question is worth 10 points, and the answer to each practical question is worth 8 points.
Evaluation criteria for the theoretical question of the examination paper:
10 points – the answer is correct, complete, well-argued;
8-9 points – the answer is correct, detailed, but not very well argued;
6-7 points - in general, the answer is correct, but has flaws;
4-5 points – there are minor errors in the answer;
1-3 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.
Evaluation criteria for the practical question of the examination work:
8 points – the answer is correct, the calculations are completed in full;
6-7 points - the answer is correct, but not very well supported by calculations;
5 points - in general, the answer is correct, but has flaws;
3-4 points – there are minor errors in the answer;
1-2 points – there are significant errors in the answer;
0 points - there is no answer or the answer is incorrect.

The maximum number of points for an answer on the exam:
R_E = 10 points × 1 theoretical question + 8 points × 5 practical tasks = 50 points.

8.5. Calculation of the rating scale (R).

The semester component of the rating scale R_S= 50 points, it is defined as the sum of positive points received for the completion of the integral modular control work, for the completion and defense of the individual IWS and negative penalty points.
The examination component of the rating scale is equal to: R_E = 50 points.
The rating scale for the discipline is equal to: $R = R_S + R_E$ = 100 points.

8.6. Calendar control: is carried out twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements:

At the first certification (8th week), the student receives "credited" if his current rating is at least 10 points (50% of the maximum number of points a student can receive before the first certification).
At the second certification (14th week), the student receives "passed" if his current rating is at least 20 points (50% of the maximum number of points a student can receive before the second certification).

8.7. Conditions for admission to the exam and determining the grade.

A necessary condition for a student's admission to the exam is the completion and defense of all individual works and the student's semester rating of at least 60% of the R_S, i.e. at least 30 points. Otherwise, the student must do additional work and improve his rating.
The total rating of the R student is defined as the sum of the semester rating of the student R_S and the R_E points received on the exam. The grade is assigned according to the value of R according to the table. 1.

Table 1

Total rating R_D	**Grade**
95-100	excellent
85-94	very good
75-84	good
65-74	satisfactory
60-64	enough
$R_D \leq 59$	unsatisfactory
$r_C < 30$ or not performed (not protected) all types of work.	not allowed

8.8 Additional information on the discipline (educational component)

A copy of the typical exam tickets issued for each semester control is given in Appendix 1.

Working program of the academic discipline (syllabus):

Compiled by DcS., prof. Legeza V.P.

Adopted by Computer Systems Software Department (protocol № 12 from 26.04.23)

Approved by the Faculty Board of Methodology (protocol № 10 from 26.05.23)

Appendix 1

TYPICAL EXAMINATION TICKET

FROM THE DISCIPLINE "MATHEMATICAL ANALYSIS-2"

FOR SEMESTER CONTROL (SECOND SEMESTER)

<table>
<tr><td colspan="4">Igor Sikorsky Kyiv Polytechnic Institute</td></tr>
<tr><td>Educational and qualification level Bachelor Specialty 121 "Software Engineering"</td><td>Department of software of computer systems

2022 – 2023 education year</td><td>EXAMINATION TICKET No. 1 from the academic discipline

MATHEMATICAL ANALYSIS-2
Second semester</td><td>I approve
Chief Department of software of computer systems
____
(signature)
DcS., Assoc.prof.
E.S. Sulema

Prot. No. 13 dated 06/22/2022</td></tr>
<tr><td colspan="4">Examination theoretical questions</td></tr>
<tr><td colspan="4">1. Number series. Sufficient signs of convergence of positive numerical series. Cauchy's integral and radical sign (with proof). An example of the application of Cauchy's integral sign.</td></tr>
<tr><td colspan="4">Practical tasks of various types</td></tr>
<tr><td colspan="4">2. The function $f(x,y)$ is defined as follows:
$$f(x,y)=\begin{cases}\dfrac{x^2y}{x^2+y^2}, & x^2+y^2\neq 0;\\ 0, & x^2+y^2=0.\end{cases}$$
It is necessary to find out: 1). Is this function continuous? 2). Do its partial derivatives exist on the entire plane OXY ? 3). Are there points of the plane OXY at which the continuity of the partial derivatives of this function is broken? Justify the answers.</td></tr>
<tr><td colspan="4">3. Find the extrema of the function $z=x^4+y^4-2x^2+4xy-2y^2$ and set their type.</td></tr>
<tr><td colspan="4">4. Solve first-order DE using the method of variation of an arbitrary constant: $y'+\dfrac{y}{x}=x^2y^4$.</td></tr>
<tr><td colspan="4">5. Using the results of the theorem on superposition of solutions, find the partial and general solutions of the second-order LIDE: $y''-5y'+6y=12e^{-x}+18x^2-7$.</td></tr>
<tr><td colspan="4">6. Find the region of convergence of a power series with complex terms: $\sum\limits_{n=0}^{\infty}\dfrac{(z+3-4i)^n}{(n+3)^2\,5^n}$.</td></tr>
</table>

Lecturer of the academic discipline, prof. ____________ V.P. Legeza