Національний технічний університет України
«КИЇвСький ПОлІтЕХНІЧнИЙ ІНСтИТУт
імені ІГОРЯ СІкОРСьКОГО»

Computer Systems Software
Department

# LINEAR ALGEBRA AND ANALYTICAL GEOMETRY <br> Syllabus 

| Requisites of the Course |  |
| :---: | :---: |
| Cycle of Higher Education | First cycle of higher education (Bachelor's degree) |
| Field of Study | 12 Information Technologies |
| Speciality | 121 Software engineering |
| Education Program | Software Engineering of Multimedia and Information Retrieval Systems |
| Type of Course | Normative |
| Mode of Studies | full-time |
| Year of studies, semester | 1 year of training, 1 semester |
| ECTS workload | 3 credits (ECTS). Time allotment - 120 hours, including 36 hours of lectures, 36 hours of classroom work and 48 hours of self-study. |
| Testing and assessment | Credit, modular control work, calendar control, computation work |
| Course Schedule | http://rozklad.kpi.ua/ |
| Language of Instruction | English |
| Course Instructors | Lecturer: senior teacher, Sushchuk-Slyusarenko Victoria Igorevna, sushchuk@pzks.fpm.kpi.ua; viss_kiev_58@ukr.net Practical classes: senior teacher, Sushchuk-Slyusarenko Victoria Igorivna |
| Access to the course | https://classroom.google.com/c/MzgyODE1NTExOTg3, код курсу: apupkyk |

## 1. Course description, goals, objectives, and learning outcomes

Studying the discipline "Linear Algebra and Analytical Geometry" allows students to develop the competencies necessary for solving practical problems of professional activity related to data analysis, information processing and the use of modern information technologies.

The goal of the discipline "Linear Algebra and Analytical Geometry" is to provide students with basic knowledge of the basics of matrix calculus, vector algebra, algebra of operators, analytic geometry on the plane and in space; the ability to solve various problems according to the classical scheme, the ability to apply the acquired knowledge to the solution of applied problems that arise in practice, the ability to construct curves and surfaces to be studied.

The subject of the discipline "Linear algebra and analytical geometry" is the methods and algorithms of linear algebra and analytical geometry used in information technologies.

The study of the discipline "Linear Algebra and Analytical Geometry" contributes to the formation of general competences (GC) in students, necessary for solving practical tasks of professional activity, related to the development, improvement and support of intelligent information systems for processing multimedia data: GC01 Ability to abstract thinking, analysis and synthesis.
GC02 Ability to apply knowledge in practical situations
GC06 Ability to search, process and analyze information from various sources.

The study of the discipline "Linear Algebra and Analytical Geometry" contributes to the formation in students of the following program learning outcomes (PLO) according to the educational program:
PLOO5 Know and apply relevant mathematical concepts, methods of domain, system and object-oriented analysis and mathematical modeling for software development.
PLO11 Select initial data for design, guided by formal methods of description of requirements and modeling. PLO25 To know and be able to use fundamental mathematical tools when constructing algorithms and developing modern softw

## 2. Prerequisites and post-requisites of the course (the place of the course in the scheme of studies in accordance with curriculum)

The study of the discipline "Linear Algebra and Analytical Geometry" takes place in the first semester.
The theoretical knowledge and practical skills obtained during the mastering of the discipline "Linear Algebra and Analytical Geometry" ensure the successful mastering of the discipline "Mathematical Analysis" and the implementation of course and diploma projects of the curriculum for bachelors in the specialty 121 Software Engineering.

## 3. Content of the course

The discipline "Linear Algebra and Analytic Geometry" involves the study of the following topics:
Topic 1. Basics of matrix calculation.
Topic 2. Fundamentals of vector algebra.
Topic 3. Systems of linear algebra.
Topic 4. Analytical geometry on the plane.
Topic 5. Analytical geometry in space.
Topic. 6. Lines of the second order.
Topic 7. Surfaces of the second order.
Topic 8. Linear spaces. Linear subspaces.
Topic 10. Linear operators.
Topic 11. Eigenvectors and eigennumbers of a linear operator.
Topic 12. Self-adjoint operators. Orthogonal matrices and operators.
Topic 13. Quadratic forms. Application of quadratic forms to determine the type of second-order curve or surface.
Modular control work
Credit

## 4. Coursebooks and teaching resources

## Basic literature:

Інформаційні джерела з фонду Бібліотеки КПІ : https://opac.kpi.ua/F

1. Rudnyeva G. V. Elements of linear algebra and analytic geometry : textbook / G. V. Rudnyeva; National

Technical University "Kharkiv Polytechnic Institute". - 2nd rev., expanded ed. - Kharkiv : Panov A. M., 2020. - 236
p. URL: http://repository.kpi.kharkov.ua/bitstream/KhPI-

Press/47954/1/Book 2020 Rudnyeva Linear algebra.pdf
2. Higher Mathematics: Analytic Geometry and Linear Algebra. Problems solving and variants of typical calculations. Vol. I. / Edited by Dr.Sci.Tech Kurpa L.V. - Kharkiv: NTU "KhPI", 2006.

## 5. Methodology

| № | Type of training session | Description of the lesson |
| :---: | :---: | :---: |
| Topic 1. Basics of matrix calculus. |  |  |
| 1 | Lecture 1. Basics of matrix calculus: definitions, operations with matrices | Types of matrices. Operations on matrices. Determinant and minors of the matrix. Properties of determinants. Scalar form of linear dependence and independence of a system of vectors. Determinant of the matrix. Matrix rank. Inverse matrix. <br> Task on self-study: item 6, number 1, 37. |
| 2 | Practical lesson 1. Basics of matrix calculus: definitions, operations with matrices | Independent work \#1. <br> Task on self-study: item 6, number 2. |
| 3 | Lecture 2. Basics of matrix calculus: rules for calculating determinants of higher orders. Matrix rank. Inverse matrix. | Determinant of a matrix: rules for calculating determinants. Matrix rank. Inverse matrix. <br> Task on self-study: item 6, number 3, 37 |
| 4 | Practical lesson 2. Inverse matrix. | Independent work \#2. <br> Task on self-study: item 6, number 4. |
| Topic 2. Fundamentals of vector algebra |  |  |
| 5 | Lecture 3. Fundamentals of vector algebra. Actions with vectors. | Linear operations on vectors, types of vectors, linear dependence of vectors, basis, calculations in coordinates. Task on self-study: item 6, number 5, 38 |
| 6 | Practical lesson 3. Visual space vectors, actions with vectors. | Independent work \#3. <br> Task on self-study: item 6, number 6. |
| 7 | Lecture 4. Basics of vector algebra: products of vectors | Products of vectors: scalar, vector, mixed and their application; linear spaces. <br> Task on self-study: item 6, number 7, 38 |
| 8 | Practical lesson 4. Products of vectors | Independent work \#4. <br> Task on self-study: item 6, number 8. |
| Topic 3. Systems of linear algebraic equations: Heterogeneous system of linear algebraic equations. General and partial solutions. Matrix method of solving systems of linear algebraic equations. |  |  |
| 9 | Lecture 5. Systems of linear algebraic equations. Defined systems | Types of systems. Methods of solving deterministic systems: Kramer's rule, matrix method, Gauss method. <br> Tasks on self-study: item 6, number 9, 39 |
| 10 | Practical lesson 5. Defined systems of linear algebraic equations. | Independent work \#5. <br> Task on self-study: item 6, number 10. |
| 11 | Lecture 6. Homogeneous and heterogeneous systems of linear algebraic equations. | Gauss method. A homogeneous system of linear algebraic equations. Fundamental system of solutions. The KroneckerCapelli theorem. <br> Task on self-study: item 6, number 11, 39 |
| 12 | Practical lesson 6. <br> Homogeneous and <br> heterogeneous systems of | Independent work \#6. <br> Task on self-study: item 6, number 12. |


|  | linear algebraic equations. |  |
| :---: | :---: | :---: |
| Topic 4. Analytical geometry on the plane |  |  |
| 13 | Lecture 7. Analytical geometry on the plane | The equation of a straight line, a bundle of straight lines. The main types of tasks. Dividing the line segment in this ratio. Orthogonal projection of a vector onto a direction <br> Task on self-study: item 6, number 13, 40 |
| 14 | Practical lesson 7. Analytical geometry on the plane | Independent work \#7. <br> Tasks on self-study: item 6, number 14. |
| Topic 5. Analytical geometry in space. |  |  |
| 15 | Lecture 8. Analytical geometry in space. The equation of the plane. | Plane. Vector and general equation of the plane. Study of the general equation of the plane. The equation of the plane in segments on the coordinate axes. The equation of the plane passing through the three given points. The equation of a plane passing through a given point parallel to two given vectors. The equation of a plane passing through two given points parallel to a given vector. The distance from the point to the plane. Tasks on self-study: item 6, number 15, 41 |
| 16 | Practical lesson 8. The equation of a plane in space. The main types of tasks. | Independent work №8. <br> Tasks on self-study: item 6, number 16. |
| 17 | Lecture 9. Analytical geometry in space. The equation of a straight line. The relative location of the straight line and the plane.. | Canonical and parametric equations of a straight line in threedimensional space. The angle between two straight lines in space. The condition that two straight lines belong to the same plane. The distance from a point to a straight line in threedimensional space. Straight and plane. The angle between a straight line and a plane. Conditions of parallelism and perpendicularity of a line and a plane. The point of intersection of a straight line with a plane. A bundle of planes. The equation of a plane passing through a line parallel to another line. The distance between two passing straight lines. <br> Tasks on self-study: item 6, \#17, 41. |
| 18 | Practical lesson 9. Analytical geometry in space. The equation of a straight line. The relative location of the straight line and the plane. | Independent work №9. <br> Tasks on self-study: item 6, number 18. |
| Topic 6. Lines of the second order |  |  |
| 19 | Lecture 10. Lines of the second order | Ellipse, hyperbola, parabola: equations, properties <br> Tasks on self-study: item 6, number 19, 42 |
| 20 | Practical lesson 10. Lines of the second order | Independent work №10. <br> Tasks on self-study: item 6, number 20. |
| Topic 7. Surfaces of the second order |  |  |
| 21 | Lecture 11. Surfaces of the second order | Ellipsoids, hyperboloids, paraboloids, cylindrical and conical surfaces. <br> Tasks on self-study: item 6, number 19, 42 |
| 22 | Practical lesson 11. | Independent work №11. |


|  | Surfaces of the second order: ellipsoids, hyperboloids, paraboloids, cylindrical and conical surfaces. | Tasks on self-study: item 6, number 22. |
| :---: | :---: | :---: |
| Topic 8. Linear spaces. Linear subspaces. |  |  |
| 23 | Lecture 12. Linear spaces. Linear subspaces. | Definition; properties; linear dependence; basis of linear space; transformation of coordinates when changing the base. Definition of linear subspace, examples; intersection and sum of linear subspaces; rank of the system of vectors; linear shells. Tasks on self-study: item 6, number 21, 43 |
| 24 | Practical lesson 12. <br> Linear spaces. Linear subspaces. | Independent work №12. <br> Tasks on self-study: item 6, number 24. |
| Topic 9. Euclidean spaces |  |  |
| 25 | Lecture 13. Euclidean spaces | Definition of Euclidean space; the Cauchy-Buniakovsky inequality; standardized spaces; orthogonal and orthonormal bases; Gram-Schmidt orthogonalization process. Tasks on self-study: item 6, number 23, 43 |
| 26 | Practical lesson 13. <br> Euclidean spaces | Independent work №13. <br> Tasks on self-study: item 6, number 26. |
| Topic 10. Linear operators. |  |  |
| 27 | Lecture 14. Linear operators. | Definition and examples of linear operators; linear operator matrix; transformation of the linear operator matrix when moving to another basis. <br> Tasks on self-study: item 6, number 25, 43 |
| 28 | Practical lesson 14. <br> Linear operators. | Independent work №14. <br> Tasks on self-study: item 6, number 28. |
| Topic 11. Eigenvectors and eigenvalues of the matrix of a linear operator. |  |  |
| 29 | Lecture 15. Eigenvectors and eigenvalues of the matrix of a linear operator. | Characteristic equation of the operator; eigenvectors of a linear operator, their properties. <br> Tasks on self-study: item 6, number 27, 44 |
| 30 | Practical lesson 15. <br> Eigenvectors and eigenvalues of the matrix of a linear operator. | Independent work №15. <br> Tasks on self-study: item 6, number 30. |
| Topic 12. Self-adjoint operators. Orthogonal matrices and operators. |  |  |
| 31 | Lecture 16. Self-adjoint operators. Orthogonal matrices and operators. | Definition of self-adjoint operator; eigenvectors of the selfadjoint operator; invariant subspaces of the self-adjoint operator. Definition of orthogonal operator; transition matrices in Euclidean space; reduction of a symmetric matrix to a diagonal form. <br> Tasks on self-study: item 6, numbers 31, 45 |
| 32 | Practical lesson 16. <br> Self-adjoint operators. Orthogonal matrices and operators. | Independent work №16. <br> Tasks on self-study: item 6, number 32. |

Topic 13 Quadratic forms. Application of quadratic forms to determine the type of second-order curve or surface.

| 33 | Lecture 17. Application of <br> quadratic forms to determine <br> the type of second-order curve <br> or surface. | Definition of quadratic form; canonical form of quadratic form; <br> orthogonal transformations of quadratic forms; Sylvester's <br> criterion. Reduction of the second-order curve equation to the <br> canonical form <br> Tasks on self-study: item 6, numbers 33, 46 |
| :--- | :--- | :--- |
| 34 | Practical lesson 17. <br> Quadratic shapes. Study of <br> quadratic forms for sign <br> determination | Independent work No8. <br> Tasks on self-study: item 6, number 34. |
| 35 | Practical lesson 18. | Test. |
|  |  | Modular control work |

## 6. Self-study

The discipline "Linear Algebra and Analytical Geometry" is based on independent preparations for classroom classes on theoretical and practical topics.

| № | The name of the topic submitted for independent processing | Number of hours | literature |
| :---: | :---: | :---: | :---: |
| 1 | Preparation for the lecture 1 | 1 | 1-2 |
| 2 | Preparation of a practical lesson 1 | 1,5 | 1-2 |
| 3 | Preparation for the lecture 2 | 1 | 1-2 |
| 4 | Preparation for the lecture 3 | 1 | 1-2 |
| 5 | Preparation of a practical lesson 2 | 1,5 | 1-2 |
| 6 | Preparation for the lecture 4 | 1 | 1-2 |
| 7 | Preparation for the lecture 5 | 1 | 1-2 |
| 8 | Preparation of a practical lesson 3 | 1,5 | 1-2 |
| 9 | Preparation for the lecture 6 | 1 | 1-2 |
| 10 | Preparation for the lecture 7 | 1 | 1-2 |
| 11 | Preparation of a practical lesson 4 | 1,5 | 1-2 |
| 12 | Preparation for the lecture 8 | 1 | 1-2 |
| 13 | Preparation for the lecture 9 | 1 | 1-2 |
| 14 | Preparation of a practical lesson 5 | 1,5 | 1-2 |
| 15 | Preparation for the lecture 10 | 1 | 1-2 |
| 16 | Preparation for the lecture 11 | 1 | 1-2 |
| 17 | Preparation of a practical lesson 6 | 1,5 | 1-2 |
| 18 | Preparation for the lecture 12 | 1 | 1-2 |
| 19 | Preparation for the lecture 13 | 1 | 1-2 |
| 20 | Preparation of a practical lesson 7 | 1,5 | 1-2 |
| 21 | Preparation for the lecture 14 | 1 | 1-2 |


| 22 | Preparation for the lecture 15 | 1 | $1-2$ |
| :--- | :--- | :--- | :--- |
| 23 | Preparation of a practical lesson 8 | 1,5 | $1-2$ |
| 24 | Preparation for the lecture 16 | 1 | $1-2$ |
| 25 | Preparation for the lecture 17 | 1 | $1-2$ |
| 26 | Preparation for modular control <br> work | 7 | $1-2$ |
| 27 | Preparation for the test | 15 | $1-2$ |
| 28 | Preparation of calculation work | 15 | $1-2$ |
| In total | 66 |  |  |

Policy and Assessment

## 7. Course policy

- Attendance at lectures and practical classes is mandatory.
- Rules of conduct in the classroom: activity, respect for those present, turning off the phones.
- Adherence to the policy of academic integrity.
- Rules for the protection of settlement work: the work must be done according to the option of the applicant, which is determined by his number in the group list.


## 8. Monitoring and grading policy

## 1) Points for answers in practical classes

The maximum number of points for practical lesson No. 1-2 points;
Evaluation criteria:
2 points - the answer is correct;
1 point - there are inaccuracies in the answer, but in general the answer is correct;
0 points - there is no answer or the answer is incorrect.
The maximum number of points for practical classes \#2-17-3 points;
Evaluation criteria:
3 points - the answer is correct;
2-1 points - there are inaccuracies in the answer, but in general the answer is correct;
0 points - there is no answer or the answer is incorrect.
The maximum number of points for answers in practical classes:
2 points $\times 1$ class $+3 \times 16$ classes $=50$ points.

## 2) Points for the modular control work

The modular control work is carried out after the completion of the study of the discipline.
Evaluation criteria:
30-25 points - the solution is correct;
24-18 points - the decision has flaws;
17-10 points - there are errors in the decision, but the course of the decision is correct;
$9-0$ score - no decision or incorrect decision.
The maximum number of points for modular control:
30 points $\times 1$ practical problem $=30$ points.

## 3) Points for camputation work

The calculation work consists of 20 problems with 1 point for each problem.
Evaluation criteria for each task:
1 point - the answer is correct, complete, well-argued;
0.5 points - in general, the answer is correct, but has shortcomings;

0 points - there is no answer or the answer is incorrect.
The maximum number of points for calculation work: $20 \times 1=20$ points.
Calculation of the rating scale (R).
The rating scale for the discipline is equal to:
$R=R c=R p r a k+R m k r+R p \pi=50$ points +30 points +20 points $=100$ points.
Calendar control: is conducted twice a semester as a monitoring of the current state of fulfillment of the syllabus requirements.
At the first certification (8th week), the student receives "credited" if his current rating is at least 10 points ( $50 \%$ of the maximum number of points a student can receive before the first certification).
At the second certification (14th week), the student receives "passed" if his current rating is at least 25 points (50\% of the maximum number of points a student can receive before the second certification).

Semester control: credit
Conditions for admission to semester control:
If the semester rating $(R C)$ is at least 60 points and the calculation work is accepted, the student receives credit "automatically" according to the table (Table of correspondence of rating points to grades on the university scale). Otherwise, he has to perform the final control work.

Completion of calculation work is a necessary condition for admission to credit control work.
If the student does not agree with the "automatic" grade, he can try to improve his grade by writing a credit test, while his points received for the semester are kept, and the better of the two grades received by the student is assigned ("soft" grading system) .The final performance score or the results of the Fail/ Pass Test are adopted by university grading system as follows:

| Score | Grade |
| :---: | :---: |
| $100-95$ | Excellent |
| $94-85$ | Very good |
| $84-75$ | Good |
| $74-65$ | Satisfactory |
| $64-60$ | Sufficient |
| Below 60 | Fail |
| Course requirements are not met | Not Graded |

## 1. Additional information about the course

In the case of a semester rating <60 points, the student is given the right to increase the score by writing an additional test.

## Syllabus of the course

Is designed by senior teacher, Sushchuk-Slyusarenko Victoria Igorevna
Adopted by Computer Systems Software Department (protocol № 12 from 26.04.23)
Approved by the Faculty Board of Methodology (protocol № 10 from 26.05.23)

